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WAVE-LENGTHS IN THE SECONDARY SPECTRUM OF HYDROGEN

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ABSTRACT

With a specially constructed tube as a source, *wave-lengths of 3064 lines between 3394 and 8902 I. A. were measured.* Of these, 95 strong lines between 4171 and 6527 A were photographed by means of a *Fabry-Perot interferometer* and auxiliary concave-grating spectrograph, and the wave-lengths measured in comparison with neon lines. For 61 of these the probable error is 0.0012 or less; for none of them is it greater than 0.0035 A. The secondary spectrum was then photographed with a 21-foot concave grating in a Rowland Mounting, *with a dispersion of 2.63 A/mm and a theoretical resolving-power of 80,000 in the first order.* The wave-lengths of the remaining 2969 lines were obtained wherever possible by interpolation between the 95 hydrogen lines measured with the interferometer. Where no such hydrogen standards existed, the spectrum of iron was used as a standard. The mean error of most of the lines thus measured is less than 0.01 A.

It is believed that *many uncertainties* due to lack of resolution in previous measurements of the spectrum *have been eliminated.* Our measures show good agreement with those of Merton and Barratt, and Allibone. The wave-lengths are systematically higher than Poettker's by about 0.24 A.

The secondary spectrum of hydrogen is probably the most complicated of those many-lined spectra which are attributed to the excitation of the molecule, and in consequence it has thus far defied attempts at complete analysis. Many qualitative experiments have been performed with a view to this analysis, and the usual method in these experiments has been to show the effect of different kinds of excitation. But with the exception of the comprehensive work of Merton and Barratt,[1] no attempt at very accurate measurements of wave-lengths with high dispersion had been made at the time we

[1] *Philosophical Transactions of the Royal Society*, A, **222**, 369, 1922.

began the work reported on in this paper. Since then, several observers[1] have published the results of their measures for different regions of the secondary spectrum observed under different conditions.

It is believed that a table containing as many wave-lengths as it is practicable to obtain, measured with as great accuracy as possible, on the international scale, will be of assistance in the analysis of this spectrum.

SOURCE OF THE SPECTRUM

After many trials, which were carried out by one of the writers,[2] the following type of discharge tube was adopted. Except for refilling, the same tube has been used throughout the work. A tube of 8-mm inside diameter and 30 cm between cylindrical aluminum electrodes was silvered on the outside, to concentrate as much light as possible toward the window at one end. The inside surface of the tube was ground with emery to procure the "white stage" described by Wood. It is possible that the advantage of silvering was partly lost on account of the ground surface inside. Electrolytically prepared hydrogen, passed through drying tubes of calcium chloride and phosphorous pentoxide, was used for filling the tube, after repeated flushing. The most satisfactory pressure was difficult to obtain. If the tube was sealed off while it showed the most intense secondary spectrum, the hydrogen within the tube soon became occluded. After several trials it was found best to seal the tubes off at a somewhat higher pressure, whereupon the pressure would decrease during the first few hours of operation until it reached a steady state. From the character of the discharge it is estimated that the pressure when sealed off was somewhat higher than 0.1-mm Hg. In addition to the use of liquid air for excluding impurities, it was found that preliminary operation of three or four hours cleared up any traces of CO_2 which might have been present. This fact is also reported by Merton and Barratt. Comparisons with spectra of CO_2, N_2, and O_2 show that no lines of these gases were present in our photographs.

The tube was water cooled during operation by immersion in a tank of running water, the window projecting through a rubber dia-

[1] Tanaka, *Proceedings of the Royal Society*, A, **108**, 592, 1925; Sandeman, *ibid.* p. 607, 1925; Allibone, *ibid.*, **112**, 196, 1926; Deodhar, *ibid.*, **113**, 420, 1926–27.

[2] K. O. Lee, *Wave-Lengths in the Spectrum of Hydrogen*. Thesis.

phragm. Excitation was by means of a 1-kw Thordarson radio transformer, giving a potential difference of about 1500 volts across the tube and a current of 50 ma. This tube is very satisfactory for the production of the secondary spectrum, the Balmer lines appearing about the same in intensity as the brightest portions of the secondary spectrum.

OBSERVATIONS AND MEASUREMENTS

The first measurements were made on 95 lines between 4171 and 6527 A obtained with a Fabry-Perot interferometer and a concave grating in a Wadsworth mounting for auxiliary dispersion. The neon lines were used as standards. Etalons of 5.0, 7.5, 10.0, and 15.0 mm were used. The reductions were made by the well-known relations

$$2e=n_1\lambda_1=n_2\lambda_2=\text{etc.},$$

and

$$E=\frac{D^2}{D_k^2-D_{k-1}^2},$$

where e is the thickness of the etalon, n is the order of interference at the center of the ring system, E is the fractional part of the order of interference, and D is the diameter of a bright ring. For 61 of these lines the probable error is 0.0012 A or less, for 18 it is between 0.0012 and 0.0020 A, and for 16 lines it is between 0.0020 and 0.0035 A. The hydrogen lines thus measured are starred in the table of wave-lengths. These lines were then used as standards for measuring by interpolation the other lines, which were obtained by the use of a 21-foot concave grating in a Rowland mounting. The grating had 14,438 lines per inch, and over $5\frac{1}{2}$ inches of ruled surface. The dispersion was about 2.637 A per millimeter in the first order, and the theoretical resolving power about 80,000. Examination of close pairs of lines show that the actual resolving power in the first order was about 45,000, making more accurate measures of the hydrogen spectrum possible than heretofore. For the stronger lines, the second order could be used. On every plate the spectrum of a Pfund arc, operated according to the latest specifications of the International Committee on Wave-Lengths, was also photographed. Although iron lines were not used as standards of comparison where hydrogen lines measured with the interferometer were available, the iron spec-

trum was useful in bridging the gaps where the hydrogen standards were far apart. For wave-lengths longer than 6600, and shorter than about 7000 A, the first-order iron spectrum was used for direct comparison, while for wave-lengths longer than 7000 A the second-order iron spectrum was compared with the first-order hydrogen lines. Exposure times ranged from an hour or two for the strongest parts of the visible spectrum to seventy-two hours for the extreme red. Two plates were obtained with the latter exposure time, but each had suffered mechanical shifts due to jars in the building. These were consequently discarded, although they proved valuable in checking the positions of many faint lines, and in determining the relative intensities of lines. Many exposures of about forty hours' duration were successful. For the standards determined by means of the interferometer, the number of plates measured ranged from 3 to 17. For the remaining lines, the number of plates varied from 1 to 5, by far the greater number of lines being based on more than 3 plates. All of the lines measured on only 1 plate were faint, and none were included whose reality was not certain.

For the region below 5000 A, ordinary Eastman 40 plates were used, while Astronomical Green Sensitive plates were found useful from 5000 to about 5800 A. For longer wave-lengths, Wratten and Wainwright Panchromatic plates, sensitized with ammonia, and Eastman 40's, bathed with dicyanin and sensitized with ammonia before use, were satisfactory. For the extreme red, plates dyed with neocyanin and sensitized with ammonia just before use were found to give the best results.

It is obvious that these measurements of some 3000 lines vary enormously in accuracy, making it difficult to decide how many decimal places should be retained. In order to give as complete information as possible and retain unity in the table, the values for wave-lengths smaller than 7000 A are given to thousandths of an angstrom, while those longer than 7000 A are given only to hundredths. The wave-numbers are given to a corresponding number of significant figures. A small letter *h* following the intensity indicates that the line is diffuse and possibly double. In cases where the measurements of wave-lengths have a mean error between 0.01 and 0.02 A, the letter "a" is affixed to the intensity; if between 0.02 and 0.03 A,

"b" is affixed, and so on. The intensities are eye estimates, "00" indicating that the line is only just seen on the plate, while "10" is the intensity of the strongest lines. Too much reliance should not be placed on the estimates of intensity, since the plates used over the wide range of wave-lengths differ greatly in quality and sensitivity. A bracket including three wave-lengths, the middle one of which is marked "U" in the intensity column, means that on some plates this doublet is unresolved. Kayser's *Schwingungszahlen* was used to obtain the wave-numbers.

NOTE

The wave-lengths shorter than 7780 A were measured over a year ago. Observations on the region from 7300 to 8900 A were made during the spring and summer of 1927, good agreement holding in the overlapping region. Since that time and just as this paper goes to press, a contribution has appeared by A. H. Poettker[1] giving wave-lengths in the secondary spectrum of hydrogen from 7504 to 10,654 A, from photographs taken with a plane grating spectrograph having a dispersion of 9 A per millimeter. Poettker states that there is a systematic difference between his measures and those of Allibone of 0.3 A. A comparison of the overlapping region in Poettker's paper and our tables shows that there is a systematic difference of about 0.24 A, Poettker's values being lower. Since the agreement between our plates is good, the mean error of observation for most lines lying below 0.01 A, it seems hardly possible that our results, in the main, can be far from correct. This point of view is borne out by the fact that our recent observations agreed well with those of over a year ago, and that we agree with the results of Allibone. It is difficult to compare our wave-lengths with those of Allibone, since his dispersion (25 A per millimeter) is so low that many of his values are due to blends. Excluding such doubtful cases, 64 lines show a residual of Allibone–Gale, Monk, and Lee of -0.002 A.

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[1] *Physical Review*, **30**, 418, 1927.

TABLE I

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
3394.838	2	29348.03	3704.567	0	26986.01	3809.021	1	26246.05
3395.159	1	29345.28	3714.686	0	26912.55	3809.874	1	26240.18
3593.378	0	27821.09	3715.990	1	26903.10	3812.174	1	26224.34
3593.542	1	27819.78	3717.317	0	26893.50	3812.744	2	26220.42
3612.897	1	27670.75	3719.178	0a	26880.04	3813.497	1	26215.22
3616.325	1	27644.53	3719.924	0a	26874.69	3814.568	00	26207.88
3616.401	1	27643.95	3722.093	2*h*	26859.00	3815.333	1	26202.65
3618.729	1	27626.15	3724.843	0	26839.18	3816.343	1a	26195.70
3620.052	1	27616.06	3732.108	2	26786.91	3816.482	00a	26194.74
3622.161	1a	27599.99	3738.098	00	26743.98	3816.789	1	26192.64
3632.681	1	27520.06	3739.623	00	26733.08	3818.184	1	26183.07
3633.328	1	27515.16	3740.268	0	26728.47	3818.895	1	26178.15
3635.094	1a	27501.79	3741.209	1	26721.85	3819.257	1a*h*	26175.68
3637.017	1	27487.25	3744.875	00	26695.60	3819.464	1a	26174.30
3646.830	0	27413.29	3751.879	2	26645.77	3821.872	1	26157.79
3652.460	2	27371.04	3752.629	0a	26640.44	3823.086	1	26149.48
3655.673	0	27347.00	3756.590	1	26612.35	3824.940	2	26136.82
3658.664	0a	27324.60	3757.590	0	26605.28	3825.622	0	26132.16
3662.480	0a	27296.15	3759.598	0	26591.05	3829.973	00	26102.48
3662.698	0	27294.50	3762.982	1	26567.16	3831.101	1	26094.79
3663.055	1a	27291.87	3764.434	00	26556.90	3832.415	0	26085.94
3664.132	3	27283.85	3768.280	00	26529.80	3833.489	0	26078.52
3664.600	00a	27280.35	3768.726	0a	26526.65	3835.381	1a‡	26065.66
3664.911	0	27278.05	3770.230	1	26516.07	3835.860	0	26062.40
3665.242	1	27275.60	3771.500	3	26507.15	3836.443	2	26058.46
3665.907	3	27270.62	3774.301	1	26487.48	3837.609	0a	26050.53
3667.524	00	27258.61	3775.576	1	26478.52	3839.296	0	26039.06
3668.840	0a	27249.84	3776.980	1	26468.69	3840.167	0	26033.18
3669.000	0a	27247.65	3777.104	1a	26467.83	3840.548	1	26030.59
3671.970	0	27226.61	3777.500	00	26465.04	3841.450	1a	26028.48
3672.342	0a	27222.86	3782.522	0	26429.92	3842.453	00b	26017.70
3673.621	3	27213.38	3782.756	0a	26428.26	3845.395	0*h*	25997.76
3673.773	1b	27212.24	3782.966	00a	26426.78	3845.834	0	25994.82
3674.398	7	27207.60	3784.156	1	26418.48	3846.226	1	25992.16
3676.800	0a	27189.85	3787.419	1a	26395.73	3848.784	1	25974.90
3677.985	00b	27181.08	3790.962	0	26371.07	3848.928	00	25973.93
3680.526	1	27162.34	3791.403	2	26368.00	3849.304	2	25971.40
3681.343	1	27156.29	3794.464	1	26346.75	3849.700	0	25968.71
3681.963	2	27151.72	3796.062	1	26335.65	3849.988	0	25966.76
3682.291	1	27149.30	3796.595	6	26331.92	3851.266	3	25958.15
3684.313	5	27134.40	3797.128	2	26328.24	3854.489	1	25936.44
3684.463	1	27133.28	3797.518	5	26325.54	3855.480	0a*h*	25929.79
3687.304	0a	27112.40	3797.908	2†	26322.82	3856.332	0	25924.06
3688.590	0a	27102.93	3798.816	1a	26316.55	3856.549	00	25922.59
3690.143	1	27091.55	3799.032	3	26315.06	3857.782	2	25914.34
3694.295	00	27061.09	3800.045	1	26308.04	3858.662	2	25908.41
3695.146	1	27054.85	3801.616	0*h*	26297.15	3859.008	1	25906.07
3697.855	1	27035.04	3802.076	1	26294.00	3859.877	3	25900.26
3699.783	1	27020.93	3803.031	5	26287.38	3860.060	1	25899.02
3700.048	1	27019.01	3803.269	0	26285.73	3860.365	00	25897.97
3701.883	1	26905.83	3806.733	1	26261.82	3860.552	00b	25895.72
3702.112	3	26903.97	3807.971	1a	26253.29	3860.711	4	25894.65

†H_θ, 3797.900. ‡H_η, 3835.387.

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
3861.496	4	25889.38	3928.388	1	25448.42	3991.506	2	25046.08
3863.202	4	25877.96	3930.676	0a	25433.73	3991.833	0	25044.09
3864.074	1	25872.14	3933.598	2	25414.84	3993.255	5	25035.16
3864.272	1	25870.80	3934.328	00	25409.49	3993.848	3	25031.45
3865.477	2	25862.73	3935.181	1a	25404.62	3996.657	2	25013.86
3865.990	00	25859.29	3939.893	0	25374.22	3997.143	2	25010.82
3866.455	00	25856.20	3942.606	0a	25356.78	3998.254	7	25003.86
3867.076	2	25852.00	3944.272	4	25345.40	3999.260	1	24997.57
3867.991	00	25845.91	3944.818	1	25342.56	3999.821	1	24994.07
3869.155	2	25838.14	3946.108	0a	25334.27	4000.210	00	24991.64
3869.931	3	25832.96	3947.091	1	25327.97	4000.840	3	24987.70
3870.158	0	25831.43	3949.081	0	25315.20	4002.074	4	24980.01
3870.630	2	25828.29	3949.400	0a	25313.16	4002.496	0*h*	24977.38
3870.910	2	25826.42	3950.093	00	25308.72	4002.770	4	24975.67
3871.594	5	25821.86	3950.573	2	25305.63	4002.983	0	24974.33
3872.354	4	25816.82	3951.122	1	25302.13	4003.371	2	24971.92
3874.120	2	25805.03	3951.484	1	25299.81	4003.781	0	24969.36
3874.864	0	25800.10	3952.819	0	25291.28	4004.480	1a	24965.00
3877.266	1	25784.04	3955.177	1	25276.14	4004.630	0	24964.06
3878.630	1	25775.02	3957.020	0	25264.42	4004.859	0a	24962.64
3879.526	4	25769.06	3959.210	0	25250.44	4005.085	00a	24961.24
3879.726	1	25767.73	3959.603	0	25247.94	4005.398	1	24959.26
3880.133	0	25765.05	3959.955	1	25245.69	4005.492	5	24958.70
3882.045	1	25752.38	3960.188	1	25244.08	4005.943	4	24955.90
3884.144	3	25738.45	3960.758	1	25240.57	4006.110	1	24954.84
3886.218	1	25724.68	3962.328	3	25230.57	4007.316	0	24947.33
3887.388	1	25716.94	3962.561	0a	25229.10	4008.715	2	24938.63
3887.836	2	25713.98	3963.144	4	25225.38	4009.258	0a	24935.25
3888.988	3§	25706.36	3963.525	0	25222.96	4009.561	2	24933.37
3889.299	1	25704.31	3963.730	1a	25221.65	4010.012	0	24930.56
3890.700	1	25695.06	3965.090	1a	25213.00	4010.738	0	24926.04
3897.517	1	25650.11	3966.800	1	25202.13	4011.238	0	24922.93
3897.929	0a	25647.40	3968.912	1	25188.73	4011.778	0	24919.57
3898.120	1	25646.15	3971.448	1	25172.63	4012.770	0	24913.43
3899.883	1	25635.57	3974.210	1	25155.14	4013.146	1	24911.08
3900.934	00	25627.68	3974.772	3	25152.59	4013.564	00	24908.50
3902.124	0a	25619.83	3975.666	2	25145.94	4014.175	0	24904.71
3902.620	3	25616.58	3976.835	3	25138.54	4015.833	1	24894.43
3905.516	00	25597.59	3977.091	1*h*	25136.92	4016.310	1	24891.47
3907.476	2a	25584.74	3978.147	1	25130.26	4016.547	2	24889.98
3908.045	0	25581.01	3978.666	2	25126.97	4017.468	00	24884.27
3910.040	1	25567.97	3979.322	1	25122.84	4017.757	00	24882.50
3913.848	1	25543.00	3979.740	1	25120.19	4018.395	0	24878.55
3914.012	00	25542.04	3982.557	1	25102.42	4018.722	00	24876.54
3916.177	00	25527.90	3982.633	4	25101.94	4018.899	3	24875.42
3916.958	1	25522.82	3983.971	1	25093.50	4019.140	0	24873.94
3917.953	1	25516.32	3985.702	3	25082.61	4020.348	00	24866.45
3924.409	6	25474.35	3986.921	0	25074.95	4020.926	1	24862.88
3925.970	0a	25464.22	3987.363	3	25072.16	4021.339	2	24860.33
3927.191	0	25456.30	3989.170	0	25060.80	4021.568	0a	24858.92
3927.244	0	25455.31	3990.031	0	25055.40	4021.792	2	24857.55
3927.901	0	25451.72	3991.145	8	25048.41	4023.340	0	24847.97

§ H_ζ, 3889.052.

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
4024.734	4	24839.38	4065.182	00	24592.24	4093.803	0	24420.29
4026.218	2	24830.20	4065.617	3	24589.58	4094.208	1	24417.87
4026.605	3	24827.83	4066.133	1	24586.49	4094.751	0	24414.65
4027.377	3	24823.07	4066.877	9	24581.96	4095.533	3	24409.98
4028.333	8	24817.19	4067.162	1	24580.27	4095.946	1	24407.51
4028.975	00	24813.22	4068.968	00	24569.33	4096.068	1	24406.78
4030.030	00a	24806.72	4069.358	0	24566.97	4096.598	0	24403.61
4031.370	2	24798.48	4069.631	10	24565.35	4097.433	3	24398.66
4031.757	3	24796.08	4069.884	2	24563.84	4097.547	0	24397.98
4032.671	0	24790.49	4070.319	0	24561.18	4098.607	0a	24391.67
4033.777	0*h*	24783.68	4071.235	4	24555.69	4101.690	......‖	24373.34
4034.074	0	24781.84	4071.395	0	24554.67	4101.737	$H\delta$	....
4035.567	4	24772.69	4071.656	1	24553.10	4101.768	......‖	24372.88
4036.334	1	24768.00	4072.341	0	24549.00	4106.231	3	24346.40
4036.708	0	24765.68	4072.544	0	24547.79	4107.788	2	24337.16
4037.326	0a	24761.87	4072.961	3	24545.26	4109.297	2	24328.22
4038.527	2	24754.51	4074.100	4	24538.39	4109.494	0	24327.06
4039.156	1	24750.65	4074.618	00	24535.26	4110.116	2	24323.37
4039.975	00	24745.69	4075.010	00	24532.91	4110.935	1a	24318.52
4040.247	00	24743.97	4075.887	0a	24527.62	4111.055	1a	24317.12
4043.110	0a	24726.47	4076.110	00	24526.29	4113.515	2	24303.27
4043.567	5	24723.72	4076.806	0	24522.08	4115.937	00	24288.98
4044.465	1	24718.22	4076.960	0a	24521.18	4116.306	1	24286.79
4044.772	0	24716.32	4078.165	1	24513.97	4123.266	0a	24245.80
4045.487	0	24711.93	4078.500	1*h*	24511.92	4123.750	1	24242.95
4047.581	2	24699.16	4078.843	7	24509.88	4124.077	0	24241.04
4047.950	1	24696.90	4078.963	1*h*	24509.16	4129.535	2	24208.41
4048.296	1a	24694.77	4079.432	0	24506.33	4130.025	2	24206.12
4048.451	4	24693.85	4080.092	0	24502.37	4131.453	2	24197.75
4049.183	1	24689.40	4080.464	00	24500.15	4131.953	1	24194.82
4051.221	2	24676.98	4080.655	0a	24498.95	4132.695	0	24190.49
4053.511	1	24663.04	4081.478	2	24494.03	4133.381	0	24186.47
4053.990	2	24660.11	4081.579	0	24493.53	4133.995	3	24182.88
4054.383	0	24657.74	4082.383	5	24489.63	4135.005	0	24176.97
4055.278	2	24652.27	4083.341	0	24482.87	4137.438	0	24162.75
4056.409	2	24645.40	4083.838	1	24479.87	4138.190	0	24158.36
4057.152	0a	24640.91	4084.010	0	24478.85	4139.519	0	24150.60
4057.551	1	24638.48	4084.519	1	24475.79	4139.780	00	24149.08
4058.433	0	24633.14	4085.008	2	24472.87	4140.817	0	24143.04
4058.618	0a	24631.98	4085.243	3	24471.46	4142.036	1	24135.93
4058.795	00	24630.89	4085.325	0	24470.97	4142.801	0	24131.48
4058.993	00	24629.74	4086.284	0	24465.23	4143.286	0	24128.64
4059.254	3	24628.16	4087.755	9	24456.42	4143.858	0	24125.31
4059.778	0	24624.94	4088.685	1	24450.86	4145.546	2	24115.47
4060.903	0	24618.15	4088.905	1	24449.55	4146.241	2	24111.46
4061.085	1	24617.06	4089.293	00a	24447.28	4146.912	0a	24107.55
4061.300	0	24615.73	4089.689	00	24444.85	4151.573	2	24080.49
4061.410	00	24615.06	4090.484	00	24440.11	4152.164	00a	24077.06
4062.457	8a	24608.70	4090.710	0	24438.76	4152.637	2	24074.32
4062.910	1	24605.98	4091.068	00a	24436.62	4155.804	2	24055.98
4063.258	1	24603.86	4091.624	0	24433.30	4156.623	4	24051.24
4063.631	4	24601.62	4092.207	0	24429.82	4156.861	4	24049.86
4064.173	0	24598.35	4092.812	0	24426.21	4157.173	1	24048.05

‖ Owing to the nearness of H_δ, no estimate of the intensities of these lines can be given.

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
4159.302	5	24035.74	4204.887	1	23775.18	4267.032	1	23428.93
4161.490	1	24023.10	4205.098*	9	23773.97	4267.906	1a	23424.13
4161.718	1	24021.79	4205.279	1	23772.96	4271.189	0a	23406.11
4161.941	5	24020.51	4206.085	2	23768.41	4276.360	0a	23377.81
4162.364	0a	24018.06	4207.541	1	23760.19	4276.898	0	23374.88
4163.012	1	24014.32	4207.898	0	23758.16	4277.480	1	23372.69
4163.393	2	24012.12	4208.225	0	23756.32	4279.416	2	23361.12
4163.605	3	24010.90	4208.430	1	23755.16	4280.740	1*h*	23353.90
4164.440	00a	24006.09	4209.169	4	23750.98	4281.816	1	23348.02
4165.195	3	24001.74	4209.690	0a	23748.06	4282.712	0*h*	23343.15
4165.864	0	23997.88	4209.845	0	23747.18	4284.069	00	23336.74
4166.727	00a	23992.92	4210.131	5	23745.57	4285.500	0*h*	23327.96
4167.567	2	23988.08	4210.680	00a	23742.47	4286.035	0	23325.04
4168.571	2	23982.31	4211.133	00	23739.91	4287.613	0	23316.46
4169.696	0	23975.83	4211.821	1a	23736.04	4288.846	0a*h*	23309.75
4171.308*	10	23966.66	4212.036	1	23734.82	4289.200	0	23307.83
4173.846	00a	23951.98	4212.498*	10	23732.21	4289.641	3	23305.44
4174.050	00	23950.82	4214.279	0	23722.19	4290.114	2	23302.87
4174.319	1	23949.27	4216.063	0	23712.15	4290.567	0	23300.41
4174.957	0	23945.61	4216.176	0	23711.52	4292.138	1	23291.68
4175.165	6	23944.42	4216.210	0	23711.33	4292.586	0	23289.45
4177.125	10	23933.19	4216.669	1a*h*	23708.74	4292.696	1	23288.85
4177.720	5	23929.78	4219.495	2	23692.84	4294.226	1	23280.55
4178.639	0a	23924.51	4222.158	5	23677.92	4295.423	1	23274.06
4179.598	4	23919.03	4222.518	5	23675.90	4295.637	1*h*	23272.91
4179.897	0	23917.32	4223.935	5	23667.96	4298.027	0	23259.97
4180.111	5	23916.10	4224.105	0	23667.01	4299.660	0	23251.16
4180.692	1	23912.77	4224.503	5	23664.78	4300.840	0	23244.75
4182.170	8	23904.31	4225.546	0	23658.94	4303.423	4	23230.80
4182.674	0	23901.43	4226.736	0	23652.27	4303.877	3	23228.36
4184.400	0a	23891.58	4227.377	2	23648.69	4303.967	1	23227.87
4184.690	0	23889.91	4228.812	1	23640.67	4305.818	1	23217.88
4187.730	0	23872.58	4231.002	1	23628.43	4306.276	3	23215.41
4188.229	1	23869.73	4232.670	2	23619.12	4307.495	0	23208.85
4188.712	0a	23866.99	4232.892	1	23617.88	4307.986	0	23206.20
4189.091	00	23864.83	4233.407	4	23615.01	4308.635	2	23202.73
4189.350	00a	23863.34	4233.818	3	23612.71	4311.017	00	23189.89
4189.462	2	23862.13	4237.545	1	23591.95	4311.701	1	23186.21
4192.107	1	23847.66	4242.470	0	23564.55	4312.394	0	23182.48
4192.869	0	23843.31	4243.125	1	23560.92	4312.873	2	23179.90
4193.875	0a	23837.59	4243.326	2	23559.80	4314.162	0a	23172.98
4194.016	1	23836.80	4245.533	0	23547.55	4315.762	0	23164.38
4194.305	1	23829.48	4246.691	2	23541.14	4316.956	00a	23157.98
4195.674*	6	23827.38	4246.988	0	23539.49	4318.004	1	23152.36
4198.210	3	23812.99	4253.289	3	23504.61	4318.220	2	23151.20
4198.455	00	23811.59	4254.029	1	23500.53	4320.660	00	23138.13
4198.717	1	23810.11	4255.221	0	23493.95	4323.510	0	23122.87
4199.793	5	23804.04	4256.722	1	23485.67	4327.362	1	23102.29
4200.099	1	23802.27	4260.343	0a*h*	23465.70	4327.927	5	23099.28
4200.893	0	23798.35	4261.668	1	23458.41	4329.644	1	23090.11
4200.971	3	23797.34	4262.778	1	23452.30	4330.401	1	23086.09
4202.279	1	23789.92	4264.178	00	23444.60	4331.375	0	23080.88
4202.439	1	23789.02	4265.162	1a	23439.19	4332.619	3	23074.26

*Measured with the interferometer.

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
4334.697	0a	23063.20	4398.239	1	22730.00	4432.362	1*h*	22555.02
4335.519	3	23058.82	4398.934	0	22726.41	4433.876	1	22547.31
4336.309	1	23054.62	4399.662	1	22722.65	4434.278	1	22545.27
4337.380	0	23048.93	4399.947	0	22721.18	4434.573	1	22543.76
4339.534	1	23037.49	4400.745	4	22717.07	4434.991	2	22541.66
4339.817	1	23036.00	4400.830	3	22716.63	4435.885	1	22537.10
4340.470	$H\gamma$	....	4400.915	0	22716.18	4436.205	00	22535.48
4341.794	0	23025.50	4401.094	0	22715.27	4437.159	00	22530.62
4343.600	1	23015.93	4401.406	0	22713.65	4437.643	0	22528.17
4346.661	1a	22999.72	4401.645	0a	22712.42	4438.206	3	22525.32
4350.423	1a	22979.81	4402.462	0	22708.20	4438.629	2	22523.16
4352.119	0	22970.93	4403.625	1	22702.20	4439.429	1	22519.11
4353.412	1	22964.05	4404.165	1	22699.42	4441.404	2	22519.10
4354.181	0	22960.00	4404.607	2	22697.15	4441.846	0	22506.86
4354.540	3	22958.10	4406.865	00	22685.51	4442.852	2	22501.76
4355.785	0a	22951.54	4407.143	0	22684.08	4443.461	0*h*	22498.68
4357.045	00a	22944.90	4409.924	00a	22669.78	4444.213	3	22494.87
4358.537	00	22937.06	4410.478	2	22666.93	4445.246	7	22489.64
4361.915	0a	22919.29	4410.618	3*h*	22666.21	4445.634	1	22487.68
4364.921	0	22903.51	4412.253*	8	22657.81	4446.815	0	22481.71
4366.650	1	22894.44	4412.484	2	22656.11	4447.553	9	22477.97
4367.726	1	22888.79	4413.499	2	22651.41	4447.932	6	22476.06
4368.491	0	22884.79	4414.218	4	22647.73	4449.104	0	22470.15
4368.988	00a	22882.19	4414.536	1	22646.10	4449.502	0	22468.14
4370.278	0	22875.43	4414.994	3	22643.75	4449.913	6	22466.06
4370.766	2	22872.87	4416.217	2	22637.48	4450.350	0	22463.85
4371.990	1	22866.48	4416.480	1	22636.12	4450.816	5	22461.50
4375.544	0	22847.90	4417.342	5	22631.71	4450.346	1	22463.84
4379.403	4	22827.77	4418.673	1	22624.89	4452.097	0	22455.06
4379.955	1	22824.89	4419.144	0	22622.48	4452.482	0*h*	22453.10
4381.609	1	22816.27	4419.494	4	22620.69	4452.767	3	22451.67
4382.181	1	22813.30	4419.818	1	22619.03	4453.141	3	22449.78
4383.030	0	22808.88	4420.304	4	22616.55	4455.023	0	22440.29
4383.313	0	22807.41	4420.915	1	22613.42	4455.342	1	22438.69
4383.494	2	22806.47	4421.163	2	22612.15	4455.461	0	22438.09
4384.193	1	22802.83	4422.647	1	22604.57	4455.666	2	22437.06
4384.456	1	22801.45	4422.772	1	22604.07	4455.955	2	22435.59
4385.704	00a	22794.99	4422.988	2	22602.82	4456.548	1	22432.61
4386.249	1	22792.13	4423.248	3	22601.50	4456.665	3	22432.02
4387.285	1	22786.75	4424.317	0	22596.04	4456.851	4	22431.10
4388.888	1	22778.43	4425.151	2	22591.78	4457.030	3	22430.19
4389.084	3	22777.41	4425.895	2	22587.98	4457.577	1	22427.44
4389.452	0	22775.51	4426.108	0	22586.89	4458.295	1	22423.83
4390.462	0	22770.26	4426.608	1	22584.34	4458.732	3	22421.63
4390.900	4	22768.00	4427.472	00	22579.93	4458.859	3	22420.99
4391.726	1	22763.71	4427.750	0*h*	22578.51	4459.118	2	22419.69
4392.093	2	22761.81	4428.050	00	22576.98	4459.464	1	22417.95
4392.725	00a	22758.54	4428.792	1	22573.20	4460.190	0	22414.29
4392.891	1	22757.69	4429.165	00	22571.30	4460.965*	10	22410.40
4394.346	1	22750.15	4429.297	0	22570.63	4461.499	1	22407.72
4394.753	00	22748.03	4430.256	00	22565.74	4461.704	0	22406.69
4396.893	0	22736.96	4431.378	1	22560.02	4461.914	1	22405.64
4398.081	1	22730.82	4431.494	1	22559.44	4462.781	0	22401.28

*Measured with the interferometer.

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
4463.207	0	22399.15	4495.625	0	22237.62	4524.883	1	22093.84
4463.876	1	22395.79	4496.672	1	22232.45	4525.210	0a	22092.25
4464.221	2	22394.07	4497.101	3	22230.34	4526.239	0	22087.22
4464.352	1	22393.40	4497.577	3	22227.98	4526.996	0	22083.53
4466.928	0	22380.49	4497.877	0	22226.50	4527.183	3a*h*	22082.61
4467.145	8	22379.40	4498.108*	10	22225.34	4527.516	0	22081.00
4467.748	0a	22376.38	4498.523	4	22223.30	4527.778	2	22079.72
4468.280	2	22373.72	4499.204	00	22219.94	4529.079	5	22073.37
4469.240	00	22368.91	4499.554	2	22218.21	4529.266	2	22072.46
4469.544	0	22367.39	4500.050	1	22215.76	4529.669	0	22070.50
4470.715	1	22361.53	4501.597	1	22208.13	4530.890	1	22064.55
4471.516	2	22357.53	4501.960	6	22206.34	4531.193	3	22063.07
4471.608	1	22357.05	4502.600	2	22203.18	4531.947	1	22059.41
4471.961	3	22355.31	4502.743	2	22202.47	4533.055	5	22054.01
4472.210	0	22354.06	4503.542	00	22198.54	4533.087	U	
4474.261	6	22343.81	4503.858	1	22196.99	4533.128	4	22053.66
4475.866	0	22335.79	4504.467	2	22193.99	4534.157	7	22048.66
4476.329	1	22333.48	4504.916	1	22191.78	4534.468	1	22047.14
4477.071	5	22329.79	4505.631	6	22188.25	4534.627	6	22046.37
4477.823	1	22326.03	4506.545	00	22183.74	4535.906	2*h*	22040.15
4478.987	3	22320.24	4508.338	00	22174.93	4537.403	0	22032.87
4479.393	1	22317.71	4508.793	00	22172.68	4537.731	4	22031.29
4479.692	2	22316.72	4509.103	2	22171.16	4537.880	2	22030.56
4480.141	0	22314.49	4510.904	5	22162.31	4538.311	2	22028.48
4480.725	00	22311.57	4511.070	1b	22161.50	4539.162	5	22024.83
4481.263	2	22308.89	4511.332	1	22160.21	4541.118	2	22014.85
4481.919	1	22305.63	4511.690	4	22158.45	4542.954	1	22005.96
4482.075	1	22304.85	4511.888	0	22157.47	4543.692	5	22002.39
4482.253	0	22303.96	4512.481	0	22154.57	4545.915	00	21991.63
4482.506	00	22302.72	4512.871	1	22151.66	4547.234	1	21985.24
4483.091	00	22299.80	4513.450	0	22149.81	4547.967	5	21981.71
4483.509	1	22297.72	4513.828	3	22147.95	4549.896	3	21972.38
4484.161	1	22294.48	4514.313	3	22145.57	4550.318	1	21970.34
4484.333	00	22293.62	4515.182	2	22141.31	4550.983	5	21967.13
4485.135	1	22289.63	4515.562	3	22139.45	4551.378	1	21965.23
4485.510	0	22287.77	4517.428	3	22130.30	4551.724	3	21963.56
4485.832	2	22286.17	4517.722	1	22128.86	4552.423	2	21960.18
4486.084	8	22284.92	4517.898	2	22128.00	4553.186	0	21956.51
4486.710	0	22281.81	4518.128	0	22126.88	4553.954	1*h*	21953.14
4487.103	0	22279.86	4518.660	1	22124.27	4554.158*	10	21951.80
4487.813	4	22276.33	4519.122	3	22122.01	4555.847	0	21943.69
4488.548	1	22272.69	4519.959	3	22117.91	4557.125	4	21937.52
4489.234	1	22269.29	4520.450	0	22115.51	4557.393	5	21936.14
4489.531	0	22267.82	4520.631	1	22114.63	4558.318	4	21931.77
4490.451*	9	22263.25	4521.417	3	22110.79	4558.606	4	21930.40
4490.852	1a	22261.26	4521.488	3	22110.43	4558.837	1	21929.31
4491.738	0a	22256.87	4522.314	0	22106.39	4560.206	2	21922.71
4491.975	1*h*	22255.69	4522.448	1	22105.74	4560.594	0	21920.84
4492.435	0	22253.41	4522.872	1	22104.67	4561.148	1	21918.18
4492.840	1	22251.41	4522.960	1	22103.24	4562.222	4	21913.02
4493.688	5	22247.21	4523.193	2	22102.13	4562.453	1	21911.91
4494.929	1	22241.08	4523.466	0	22100.76	4563.728	3	21905.79
4495.313	1	22239.17	4524.139	7	22097.47	4564.245	0	21903.32

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
4564.690	1	21901.17	4604.928	0	21709.81	4643.486	0	21529.54
4564.909	0a	21900.12	4605.364	2	21707.75	4643.850	1	21527.84
4565.544	3	21897.08	4607.091	1	21699.61	4644.905	0a	21522.96
4567.753	0	21886.48	4607.403	4	21698.15	4645.344	3	21520.92
4568.130*	10	21884.68	4608.564	0a	21692.67	4646.579	1	21515.19
4568.617	1a	21882.35	4609.120	00	21690.06	4648.199	0a	21507.70
4569.774	0	21876.81	4609.257	0a*h*	21689.41	4648.585	1*h*	21505.92
4570.799	1	21871.90	4613.116	4	21671.27	4649.273	0	21502.76
4571.164	0	21870.15	4613.637	0	21668.82	4649.502	1	21501.68
4571.245	0	21869.77	4614.588	3	21664.36	4650.381	1	21497.62
4572.225	2	21865.07	4616.488	00	21655.44	4650.950	0	21494.98
4572.710*	8	21862.76	4617.528	10	21650.56	4651.505	1	21492.42
4572.987	0	21861.44	4617.872	0	21648.95	4651.920	0a	21490.50
4573.524	00	21858.87	4618.126	1	21647.76	4652.999*	6	21485.52
4573.896	00	21857.09	4618.304	5	21646.92	4653.196	2	21484.61
4574.240	00	21855.83	4618.600	2	21645.54	4654.056	3	21480.64
4574.732	0	21853.10	4619.572	0a	21640.98	4654.530	1	21478.45
4575.750	1	21848.24	4620.563	1a	21636.35	4657.861	2	21463.10
4575.878	8	21847.63	4620.756	5	21635.44	4658.784	1	21458.84
4575.981	0	21847.14	4621.806	0	21630.52	4659.284	0	21456.53
4576.535	1	21844.49	4622.303	0a	21628.20	4660.099	1	21452.78
4576.732	1*h*	21843.55	4624.097	1	21619.80	4660.395	5	21451.42
4577.219	1	21841.22	4624.731	0	21616.85	4660.731	0	21449.88
4578.015	6	21837.43	4625.311	5	21614.14	4661.402	8	21446.79
4578.558	1	21834.84	4625.457	0	21613.45	4661.978	00b	21444.14
4579.122	0	21832.15	4625.584	3	21613.85	4662.811*	8	21440.31
4579.454	5	21830.56	4626.764	0*h*	21607.34	4664.634	1	21431.93
4579.579	1a	21829.97	4627.051	1	21606.01	4665.585	6	21427.56
4579.994*	10	21827.99	4627.986*	10	21601.63	4665.770	00	21426.71
4581.538	4	21820.64	4628.365	0	21599.78	4666.200	0	21424.74
4582.163	00a	21817.66	4628.708	1	21598.27	4667.083	3	21420.70
4582.592*	10	21815.61	4629.673	2	21593.77	4667.791	4	21417.44
4583.039	0	21813.48	4630.127	1	21591.65	4669.278	3	21410.61
4583.880	2	21809.48	4631.074	1	21587.23	4670.015	2	21407.23
4584.499	5	21806.54	4631.460	8	21585.45	4670.652	4	21404.31
4585.116	1	21803.61	4631.849	9	21583.62	4670.808	0	21403.60
4586.120	0*h*	21798.83	4633.606	0	21575.44	4671.305*	7	21401.31
4586.591	0	21796.60	4633.693	0	21575.04	4671.710	1	21399.47
4588.678	3	21786.69	4634.032*	10	21573.46	4671.840	0	21398.87
4588.833	0	21785.95	4634.595	5	21570.83	4672.080	1	21397.77
4589.916	1	21780.80	4634.622	U		4673.097	3	21393.11
4591.806	2	21771.84	4634.708	2	21570.31	4673.233	1	21392.50
4591.925	0	21771.28	4636.927	1	21559.99	4674.440	0	21386.97
4593.579	0	21763.44	4637.118	0	21559.10	4674.530	4	21386.56
4594.740	0*h*	21757.96	4637.400	0a*h*	21557.79	4674.958	3	21384.60
4595.166	1	21755.92	4638.012	0c	21554.94	4675.312	4	21382.99
4596.088	0*h*	21751.56	4638.465	1*h*	21553.84	4676.917	1	21375.64
4596.586	0	21749.20	4639.898	1	21546.18	4678.146	0a	21369.03
4596.694	0a	21748.69	4640.192	0	21544.82	4679.092	5	21365.71
4597.206	4	21746.27	4640.478	1	21543.49	4680.432	5	21359.59
4598.493	3	21740.19	4641.084	0	21540.67	4680.740	1*h*	21358.18
4600.168	0	21732.26	4642.668	1	21533.33	4681.354	2	21355.38
4600.856	1	21729.01	4643.248	0	21530.64	4681.643	00	21354.07

*Measured with the interferometer.

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
4681.819	00	21353.26	4735.258	0*h*	21112.28	4794.190	1	20852.77
4682.341	6	21350.86	4735.658	00a	21110.50	4795.154	1	20848.57
4683.640	1	21344.96	4737.445	0	21102.54	4796.992	4	20840.59
4683.824*	8	21344.12	4738.707	1	21096.92	4797.050	U	
4684.654	4	21340.34	4740.985*	5	21086.78	4797.104	4	20840.10
4686.142	3	21333.56	4741.812	0	21083.10	4797.754*	2	20837.28
4686.772	6	21330.69	4742.109	3	21081.77	4799.145	0	20831.24
4687.629	0	21326.79	4742.780	6	21078.81	4801.993	2a	20822.35
4688.438	2	21323.11	4743.383	5	21076.13	4802.395	1	20817.14
4690.184	7	21315.17	4743.586	1	21075.22	4804.065	2	20809.91
4691.154	1	21310.77	4744.312	0a	21071.99	4807.335	2	20795.75
4692.040	5	21306.76	4745.314	2	21067.54	4807.490	0a	20795.08
4693.425	0	21300.46	4745.968	1	21064.65	4808.783	0	20789.00
4693.808	1	21298.72	4746.610	0a	21061.79	4810.446	0a	20782.31
4697.423	1	21282.33	4750.729	1	21043.53	4810.795	00a	20780.79
4699.832	1	21271.42	4751.575	2	21039.79	4811.200	0	20779.04
4700.216	0	21269.71	4752.553	0	21035.46	4811.605	1	20777.30
4701.221	0	21265.14	4752.845	0a	21034.17	4811.934	0	20775.87
4701.723	2	21262.87	4755.405	2	21022.84	4812.806	1	20772.11
4701.891	0a	21262.11	4756.097	0	21019.78	4813.601	7	20768.69
4702.335	1	21260.10	4756.948	5	21016.02	4817.514	3	20751.81
4702.562	5	21259.07	4757.435	1	21013.94	4819.224	00a	20744.64
4705.260	3	21247.88	4758.450	2	21009.39	4819.461	1*h*	20743.43
4708.489	2	21232.31	4759.851	0a	21003.21	4822.943*	8	20728.46
4709.122	1	21229.46	4760.124	1	21002.00	4824.099	0	20723.48
4709.536*	10	21227.60	4763.844*	9	20985.60	4824.568	4	20721.47
4711.067	3	21220.69	4765.570	0	20978.00	4827.038	0	20710.87
4711.672	0	21217.97	4767.247	2	20970.62	4827.522	00	20708.79
4712.805	0b	21212.87	4767.922	0	20967.65	4827.893	00	20707.20
4713.930	6	21207.81	4768.181	1	20966.52	4830.630	2	20695.47
4714.308	1a	21206.11	4769.519	1	20960.64	4830.776	0a	20694.84
4615.344	1	21201.44	4770.610	3	20955.84	4831.563	3	20691.47
4715.925	1	21198.84	4770.910	2	20954.51	4832.792	5	20686.21
4716.122	0	21197.95	4774.272	1	20939.76	4833.371	1	20683.74
4718.810	1	21185.87	4775.864	0*h*	20932.78	4833.747	1a	20682.12
4719.043*	10	21184.82	4777.454	6	20925.81	4835.372	0	20675.17
4719.300	1a	21183.67	4780.921	5	20910.65	4836.907	1	20668.61
4720.498	1a	21178.30	4780.957	U		4838.081	0a	20663.60
4721.000	1a	21176.05	4780.981	5	20910.38	4838.242	6	20662.91
4721.542	3	21173.61	4781.781	0	20906.89	4841.521	0	20648.91
4723.032*	10	21166.94	4782.443	1a	20903.99	4841.717	1	20648.07
4723.838	1	21163.32	4782.869	1	20902.12	4842.385	3	20645.23
4724.820	4	21158.92	4783.442	0a	20899.62	4843.237	2	20641.59
4726.144	1	21153.00	4784.783	2	20894.78	4843.434	0	20640.76
4727.741	1	21146.86	4785.092	1	20892.33	4843.757	1	20639.38
4728.500	1	21142.46	4785.983	2	20888.53	4845.497	2	20631.97
4728.887	1	21140.73	4786.205	0	20887.56	4845.983	1	20629.90
4730.754	2	21132.38	4789.418	5	20873.55	4848.107	1	20620.86
4731.643	1	21128.42	4790.415	1	20869.20	4849.303*	9	20615.78
4731.922	0a	21127.17	4790.864	2	20867.24	4849.532	0	20614.80
4732.725	2	21123.59	4791.222	0	20865.69	4849.848	1	20613.46
4733.025	1	21122.25	4792.537	1	20859.96	4851.339	1	20607.12
4734.675	0*h*	21114.89	4793.913	5	20853.98	4852.985	1a	20600.14

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
4854.830	1	20592.31	4922.055	00	20311.06	4976.630	3	20088.33
4856.553*	9	20585.01	4922.450	1	20309.43	4977.171	1	20086.15
4858.390	00	20577.22	4924.018	3	20302.96	4978.256	4	20081.77
4858.754	3	20575.67	4925.233	3	20297.95	4979.115	2	20078.80
4860.108	2	20569.95	4925.782	0	20295.69	4980.479	6	20072.80
4860.806	0	20566.99	4926.261	1a	20293.69	4981.157	0	20070.07
4861.328	Hβ		4928.365	4	20285.06	4983.178	0	20061.93
4861.738	3	20563.05	4928.637	9	20283.93	4983.524	0	20060.53
4863.643	1	20554.99	4928.795	8	20283.29	4984.718	00	20055.74
4866.311	1	20543.73	4931.800	1	20270.93	4989.240	1	20037.55
4867.029	3	20540.69	4932.263	7	20269.03	4990.147	4	20033.91
4867.800	1a	20537.44	4933.515	4	20263.88	4994.089	2	20018.09
4869.451	4	20530.48	4933.842	0	20262.54	4995.214	0	10013.58
4870.289	1	20526.94	4934.241*	10	20260.90	4996.461	2	20008.60
4873.010	8	20515.48	4934.579	1	20259.51	4996.852	3	20007.03
4873.317	2	20514.19	4935.242	3	20256.79	4997.937	5	20002.69
4874.289	4	20510.10	4925.940	1	20253.93	5000.222	1	19993.55
4874.755	0	20508.15	4936.140	1a	20253.11	5003.398	9	19980.86
4875.964	6	20503.04	4936.427	1a	20251.93	5007.988	8	19962.54
4878.128	3	20493.95	4936.708	2	20250.78	5008.398	0	19960.91
4881.517	2	20479.72	4937.318	0	20248.48	5008.543	1	19960.33
4881.898	0	20478.13	4937.774	0	20246.40	5008.615	U	
4882.330	00	20476.32	4939.162	5	20240.71	5008.662	1	19959.85
4882.777	1	20474.44	4939.603	4	20238.91	5009.618	1	19956.05
4883.779	1	20470.24	4040.014	1	20237.22	5011.189	8	19949.79
4883.863	U		4941.990	2	20229.13	5013.036*	10	19942.45
4883.890	1a	20469.77	4942.546	3	20226.86	5014.473	4	19936.73
4886.406	2	20459.24	4943.612	00	20222.49	5015.069	8	19934.35
4886.895	0	20457.15	4944.823	2	20217.55	5016.339	1	19929.31
4888.478	2	20450.57	4945.802	0	20213.54	5016.496	6	19928.69
4891.269	4	20438.89	4949.536	3	20198.29	5017.127	5	19926.18
4891.940	1	20436.09	4951.210	0	20191.46	5018.753	00	19919.73
4895.882	0	20419.64	4952.585	3	20184.63	5019.717	00	19915.90
4896.470	2	20417.18	4953.160	0	20183.51	5020.744	4	19911.82
4896.893	0	20415.43	4955.004	1	20176.00	5022.678	0	19904.16
4897.475	1	20412.98	4955.492	1	20174.02	5024.963	1	19895.10
4898.873	0	20407.16	4955.763	6	20172.92	5025.878	0a	19891.49
4900.886	2	20398.78	4956.792	5	20168.72	5027.700	1	19884.28
4901.366	0	20396.79	4958.838	0	20160.41	5028.851	2	19879.73
4901.870	2	20394.70	4960.119	0	20154.79	5030.367*	9	19873.74
4903.585	1	20387.56	4960.950	2	20151.83	5032.148	1	19866.70
4906.023	1	20377.44	4965.093	2	20135.01	5034.255	1	19858.39
4906.336	4	20376.15	4966.702	1	20128.48	5039.821*	9	19836.46
4906.780	1	20374.29	4966.909	6	20127.64	5041.627	8	19829.35
4908.063	3	20368.97	4967.861	0	20123.80	5043.916	1	19820.36
4908.782*	5	20365.97	4968.626	0	20120.69	5047.715	0a	19805.44
4912.316	1	20351.33	4969.222	5	20118.28	5048.004	5	19804.30
4914.997	0	20340.22	4970.710	0	20112.25	5049.365	1*h*	19798.96
4916.273	0	20334.94	4971.762	1	20107.99	5050.420	0	19794.83
4918.385	00	20326.21	4973.310*	8	20101.74	5052.710	0	19785.84
4919.127	5	20323.15	4974.158	0a	20098.31	5054.835	2a	19777.54
4920.330	1	20318.18	4975.877	0	20091.21	5055.091*	9	19776.54
4921.200	2	20314.59	4976.210	0	20090.02	5056.250	1	19772.01

*Measured with the interferometer.

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
5057.344	0	19767.72	5135.760	0	19465.91	5199.332	1*h*	19227.90
5058.016	0*h*	19765.11	5137.109	2	19460.79	5199.707	4	19226.51
5061.732	4	19750.59	5143.420	0	19436.91	5202.027	1	19217.95
5063.742	1	19742.75	5143.567	3	19436.36	5203.023	1	19214.26
5063.878	6	19742.22	5146.340	5	19425.89	5204.325	1	19209.46
5066.326	0	19732.69	5147.761	0	19420.53	5205.068	2	19206.71
5067.475	5	19728.21	5148.440	00	19417.96	5205.501	0a	19205.12
5068.121	7	19725.70	5150.052	0	19411.89	5207.811	00	19196.60
5069.659	1	19719.71	5150.445	00a	19399.10	5208.437	0	19194.28
5069.984	0	19718.44	5153.870	6	19397.50	5211.511	0	19182.97
5070.296	1	19717.24	5154.499	0	19395.13	5213.148	1	19176.94
5072.307	2	19709.41	5155.145	0*h*	19392.71	5214.623	6	19171.51
5072.621	1	19708.20	5156.262	1*h*	19388.51	5215.602	1	19167.92
5075.442	4	19697.24	5156.842	1	19386.33	5219.865	0	19152.26
5076.136	1	19694.55	5158.164	0b	19381.36	5221.125	1a*h*	19147.64
5077.204	1	19690.44	5158.380	1	19380.54	5221.452	1	19146.47
5078.780	0*h*	19684.29	5158.836	0	19378.84	5222.596	1	19142.24
5080.494	7	19677.66	5159.831	2	19375.09	5222.854	3	19141.31
5080.956	1	19675.87	5160.525	1	19372.49	5223.540	2	19138.80
5081.433	2	19674.02	5161.476	0*h*	19368.93	5224.402	2	19135.64
5081.928	2	19672.11	5163.860	0	19359.98	5224.826	1	19134.07
5082.767	1	19668.85	5164.404	0a	19357.95	5226.771	6	19126.96
5084.842	9	19660.83	5164.720	1	19356.75	5228.085	00	19122.16
5085.500	0*h*	19658.29	5167.406	1	19346.69	5229.225	1*h*	19117.98
5089.107	0	19644.35	5168.235	4	19343.59	5229.576	0	19116.71
5090.217	1	19640.06	5168.825	1	19341.39	5229.710	0	16116.21
5092.284	1	19632.10	5169.230	00	19339.87	5230.491	00	19113.35
5093.323	0a	19628.09	5169.610	0a	19338.45	5231.710	1*h*	19108.90
5094.704	1	19622.77	5170.172	0a	19336.34	5235.982	0	19093.26
5096.214	1b*h*	19616.95	5170.320	0b	19335.79	5239.012	4	19082.27
5099.748	2	19603.36	5170.920	0	19333.54	5239.824	0	19079.30
5103.282	0	19589.79	5171.338	1	19332.35	5240.762	0	19075.90
5103.566	4	19588.70	5172.105	00	19329.12	5241.285	1	19073.98
5105.235	0a	19582.68	5172.545	00a	19327.47	5242.427	1	19069.85
5106.789	1	19576.33	5172.918	0	19326.08	5243.718	2	19065.14
5107.644	3	19573.06	5173.783	1	19322.85	5244.635	1	19061.81
5108.360	0a	19570.32	5174.702	6	19319.42	5245.604	1	19058.30
5109.307	4	19566.76	5175.253	0	19317.37	5247.362	0	19051.90
5111.738	1	19557.39	5175.833	0a	19315.19	5247.667	00	19050.80
5113.126	7	19552.08	5176.768	00	19311.20	5249.710	0	19043.38
5113.285	0	19551.47	5178.492	1	19305.28	5250.182	0	19041.67
5115.345	2*h*	19543.59	5180.583	7	19397.49	5250.332	0	19041.13
5116.943	0	19537.49	5181.316	0	19394.76	5252.562	1	19033.23
5117.390	1	19535.78	5185.125	0	19380.58	5256.610	7	19018.39
5118.961	0	19529.79	5187.427	1	19372.04	5257.118	0	19016.55
5120.131	0a	19525.33	5188.285	2	19268.84	5258.511	1	19011.51
5120.632	00	19523.42	5190.388	0	19261.02	5259.384	1	19008.37
5120.981	1	19522.09	5191.026	2	19258.66	5259.910	0a	19006.46
5122.583	4	19515.98	5193.008	1	19251.31	5261.182	7	19002.86
5127.938	1*h*	19495.90	5196.375	8	19238.85	5262.252	0	18998.00
5128.817	00a	19492.25	5197.213	4	19235.74	5263.083	1	18995.00
5132.026	2	19480.07	5197.516	1	19234.62	5263.608	1	18993.10
5134.120	1	19472.12	5198.601	0	19230.61	5263.899	2	18992.05

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
5264.284	2	18990.67	5329.042	1	18759.90	5383.760	00	18569.23
5264.684	3	18989.26	5329.628	1	18757.83	5385.508	5	18563.20
5266.045	10	18984.32	5329.931	0	18756.76	5386.195	3	18560.82
5268.237	1	18976.42	5331.245	1	18752.14	5386.746	2	18558.93
5268.875	00	18974.12	5331.614	2	18750.84	5387.530	2	18556.24
5270.412	4	18968.59	5332.153	0	18748.95	5388.166*	10	18554.05
5272.296	10	18961.78	5333.225	0	18745.18	5389.215	0	18550.43
5272.530	0	18961.46	5334.264	6	18741.52	5389.657	0	18548.91
5273.023	3	18959.19	5335.279	1	18737.96	5390.519	1	18545.94
5273.394	0a	18957.85	5336.322	1	18734.30	5391.144	3	18543.80
5273.808	1	18956.39	5336.581	5	18733.29	5391.540	1	18542.43
5274.105	0	18955.30	5337.988	0	18728.43	5392.285	6	18539.87
5276.292	1	18947.44	5339.389	2	18723.53	5393.533	0a*h*	18535.58
5277.806	1	18942.00	5340.820	4	18718.52	5394.066	0	18533.76
5278.332	2	18939.77	5343.166	4	18710.30	5394.388	1	18532.64
5279.623	0	18935.50	5344.240	1	18706.54	5394.797	2	18531.23
5280.510	0*h*	18932.31	5344.792	4	19704.60	5395.342	0	18529.37
5283.285	3	18922.37	5345.845	1	18700.92	5399.814	0	18527.75
5284.500	9	18918.02	5346.997	0	18696.89	5396.393	0	18525.75
5289.090	1a	18901.60	5350.204	0	18685.70	5398.074	1	18519.98
5289.465	1a	18900.25	5350.708	1	18683.93	5398.970	4	18516.91
5290.370	0	18897.03	5351.735	2	18780.35	5399.815	2	18514.02
5290.560	0	18896.35	5352.287	00	18678.42	5400.073	0	18513.13
5291.595	9	18892.67	5353.368	0	18674.65	5401.053	8	18509.78
5293.083	00	18887.34	5354.153	0*h*	18671.91	5401.720	1a	18507.50
5295.029	00	18880.40	5354.752	2	18669.82	5402.060	1b*h*	18506.33
5296.089	3	18876.62	5355.912	7	18665.78	5402.599	0	18504.48
5296.863	00	18873.86	5357.305	2	18660.93	5403.198	0	18502.42
5297.690	0	18870.92	5358.236	1	18657.68	5403.934	0	18499.91
5299.283	0	18865.24	5358.727	1	18655.93	5404.746	3	18497.14
5299.703	2	18863.74	5359.394	0*h*	18653.66	5405.328	5	18495.14
5302.106	2	18855.19	5361.585	0a	18646.03	5405.743	1	18493.72
5303.104	9	18851.66	5362.380	1	18643.27	5407.026	1	18489.33
5303.238	4	18851.17	5362.835	2	18641.69	5407.890	1	18486.37
5303.978	0	18848.54	5363.508	0	18639.34	5408.789	6	18483.30
5304.415	5	18846.99	5365.425	0a	18632.68	5409.692	4	18480.23
5305.565	0	18842.91	5365.902	7	18631.03	5410.219	3	18478.42
5308.614	1	18832.09	5368.355	1	18622.51	5410.790	1	18476.47
5309.036	5	18830.58	5368.831	0	18620.86	5411.454	0	18474.20
5311.623	1	18821.41	5369.384	0	18618.94	5412.662	0	18470.08
5313.168	1	18815.94	5371.896	4	18610.24	5414.575	0a	18463.55
5313.594	1	18814.43	5372.445	3	18608.34	5415.134	0a	18461.65
5313.950	00	18813.17	5373.084	2	18606.12	5415.922	0	18458.96
5314.354	2	18811.73	5373.413	2	18604.98	5417.797	6	18452.56
5315.165	0	18808.87	5374.288	1a	18601.95	5418.616	0*h*	18449.78
5315.780	0a	18806.69	5375.615	0a	18597.36	5419.893*	10	18445.44
5317.889	5	18799.23	5376.511	1	18594.27	5421.141	1	18441.19
5319.162	3	18794.74	5377.203	0	18591.87	5421.316	0	18440.59
5320.432	2	18790.26	5378.391	5	18588.77	5422.132	2	18437.81
5321.497	2	18786.49	5379.516	2	18583.85	5423.253	2	18434.00
5322.331	0	18783.54	5379.975	1	18582.32	5425.200	4	18427.39
5326.775	4	18767.87	5380.973	0	18578.84	5425.380	0a	18426.78
5327.279	0	18766.10	5381.709	0	18576.31	5425.887	8	18425.06

*Measured with the interferometer.

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
5426.320	2*h*	18423.59	5479.900	0	18243.52	5534.054	3	18064.93
5426.473	2*h*	18423.08	5481.083*	10	18239.51	5534.475	0a	18063.55
5427.100	0	18420.94	5482.852	1	18233.64	5535.980	5	18058.64
5428.100	0a	18417.55	5483.547	0	18231.32	5536.353	0	18057.42
5428.382	0a	18416.59	5484.012	1	18229.78	5537.288	3	18054.39
5428.950	2	18414.66	5485.716	2	18224.10	5537.466*	10	18053.90
5429.311	0	18413.44	5468.150	0a	18222.82	5538.657	1	18049.92
5429.854	0	18411.60	5487.286	1	18218.89	5539.963	1	18045.66
5430.150	0	18410.59	5487.971	0	18216.62	5540.599	0	18043.59
5430.871	3	18408.15	5488.286	1	18215.57	5540.938	1	18042.49
5432.318	1	18403.25	5489.475	0	18211.64	5542.263	00	18038.17
5433.108	00	18400.57	5490.163	1	18209.34	5543.112	4	18035.41
5433.606	1	18398.87	5491.253	1	18205.74	5543.496	7	18034.15
5434.822*	10	18394.77	5492.966	2	18200.05	5544.520	0	18030.83
5436.126	00	18390.35	5495.964	8	18190.12	5544.679	0	18030.31
5436.331	0	18389.66	5496.629	1*h*	18187.92	5544.771	3	18030.02
5437.806	1	18384.66	5497.709	2	18184.35	5545.507	00	18027.62
5438.777	3	18381.39	5499.213	1b	18179.38	5546.492	1	18024.36
5439.690	1a*h*	18378.30	5499.581	9	18178.16	5547.631	2*h*	18020.72
5440.188	2	18376.62	5500.335	0	18175.67	5548.095	0	18019.22
5440.705	0	18374.88	5500.482	1	18175.18	5548.403	1	18018.24
5442.213	0	18369.79	5505.522*	10	18158.55	5549.473	2	18014.73
5442.516	0	18368.75	5506.341	4	18155.85	5549.740	2	18013.87
5443.400	0	18365.78	5507.853	3	18150.87	5550.401	1	18011.73
5444.350	0	18362.57	5508.090	0	18150.08	5552.525	8	18004.84
5446.672	2	18354.75	5509.195	0a	18146.44	5553.829	1	18000.61
5447.466	1	18352.07	5509.475	0a	18145.52	5555.102	3	17996.46
5449.850	1*h*	18344.04	5510.265	1	18142.91	5555.782	0	17994.29
5452.082	1a	18336.53	5510.876	1	18140.90	5556.481	2	17992.02
5452.529	2	18335.03	5512.976	1	18134.00	5557.380	1	17989.11
5454.736	00	18327.61	5513.558	0	18132.10	5557.945	1	17987.29
5455.316	4	18325.66	5513.920	1c	18130.90	5558.311	0	17986.10
5456.685	0	18321.06	5514.599	2	18128.66	5558.924	2	17984.12
5456.983	6	18320.06	5515.220	00	18126.62	5559.714	0	17981.37
5457.905	0	18316.97	5516.559	00	18122.22	5560.239	1*h*	17979.86
5458.785	0a*h*	18314.02	5517.476	1	18119.22	5561.418	1a	17976.02
5459.598	8	18311.28	5517.991	0	18117.45	5561.741	3	17975.00
5461.400	2a	18305.25	5518.472	8	18115.94	5562.238	1	17973.40
5461.839	1*h*	18303.77	5520.881	3	18108.03	5564.506	4	17966.06
5462.304	0	18302.23	5521.831	2	18104.92	5564.884	1	17964.86
5462.990	4	18299.92	5522.639	0	18102.27	5565.509	0	17962.83
5465.192	4	18292.55	5523.965	3	18097.93	5565.650	0	17962.70
5466.457	2	18288.32	5524.152	1	18097.31	5565.780	0	17961.96
5470.104	00	18276.13	5526.315	2	18090.23	5566.193	1	17960.64
5470.530	0a	18274.70	5527.346	3	18086.52	5566.928	1a*h*	17958.25
5471.577	3	18271.27	5527.793	1	18085.38	5567.752	1	17955.60
5471.841	00	18270.32	5528.212	0	18084.02	5569.504	2	17949.95
5473.302	1	18265.45	5529.110	0a	18081.08	5571.594	2	43179.20
5474.856	5	18260.26	5529.510	1	18079.77	5572.402	1	17940.62
5476.266	2	18255.56	5529.870	2	18078.60	5573.515	00b	17937.03
5476.687	0	18254.15	5530.603	0	18076.20	5573.920	6	17935.73
5477.183	1	18252.50	5531.198	0	18074.57	5574.507	0	17933.84
5477.815	0	18250.40	5532.440	0	18070.20	5575.244	0	17931.48

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
5578.712	3	17920.33	5625.911	1	17769.98	5677.423	00	17608.81
5579.300	1	17918.43	5626.719	2	17767.43	5679.018	1	17603.80
5579.472	1	17917.88	5627.435	3	17765.17	5680.143	1	17600.32
5579.591	3*h*	17917.50	5629.790	0	17757.73	5682.079	0	17594.32
5579.963	1	17916.30	5630.620	4	17755.11	5682.507	3	17593.01
5580.740	0	17913.81	5630.991	2	17753.95	5683.080	4	17591.22
5581.059	0	17912.80	5631.815	0	17751.35	5683.744	3	17589.17
5581.904	1	17910.08	5632.190	2a	17750.16	5684.126	5	17588.00
5583.041	2	17906.43	5632.337	1	17749.72	5684.535	1	17586.73
5583.344	0a	17905.45	5634.161	3a	17743.95	5685.485	0	17583.80
5583.886	0a	17903.73	5634.457	1	17743.02	5686.249	1	17581.42
5584.384	0	17902.12	5634.807	8	17741.92	5687.459	2	17577.68
5584.908	0a	17900.44	5635.562	1	17739.54	5687.923	1	17576.22
5585.474	1	17898.62	5636.448	00	17736.76	5689.195*	10	17572.31
5585.934	1	17897.15	5637.125	00	17734.63	5689.858	0a	17570.26
5588.463	0	17889.06	5638.972	1	17728.82	5691.155	3	17566.27
5590.094	3	17883.85	5640.416	2	17724.28	5692.095	1a	17563.36
5590.531	0	17882.44	5641.521	1	17720.81	5692.465	3	17562.23
5590.964	1	17881.06	5642.068	1	17719.08	5694.140	5	17557.05
5591.420	4	17879.60	5642.717	3a	17717.06	5694.616	0	17755.59
5594.170	0	17870.80	5642.942	5	17716.35	5695.317	1	17553.43
5594.278	0	17870.46	5644.635	0	17711.04	5696.175	3	17550.78
5594.422	0	17870.00	5646.109	2	17706.41	5696.874	0a	17548.63
5595.051	0	17867.99	5647.406	0	17702.34	5697.401	2a	17547.00
5595.376	0	17866.95	5648.832	1	17697.87	5697.531	2	17546.60
5596.177	0	17864.39	5650.719	0	17691.96	5698.007	1	17545.14
5597.636*	10	17859.74	5652.175	3	17687.41	5699.206	2	17541.45
5598.322	0	17857.55	5652.569	0	17686.18	5700.644	4	17537.02
5598.852	0	17855.86	5654.462	00a	17680.25	5702.408	1	17531.60
5599.506	1	17853.78	5654.761	2	17679.32	5703.252	5	17529.00
5600.416	7	17850.87	5655.750*	9	17676.23	5703.760	4	17527.44
5601.704	3	17846.77	5657.286	1	17671.44	5705.501	0	17522.10
5604.047	1	17839.31	5657.797	2	17669.81	5708.658	2	17512.71
5604.356	1	17838.32	5659.429	1	17664.73	5709.772	5	17508.99
5604.685	3	17837.29	5659.902	0	17663.26	5712.060	1	17501.97
5609.495	1	17821.99	5660.368	0	17661.82	5713.004	2	17499.09
5611.862	1	17814.46	5660.791	1	17660.45	5713.058	U	
5612.541*	10	17812.31	5661.626	3	17657.88	5713.144	2	17498.65
5613.417	0	17809.53	5661.835	2	17657.24	5713.449	5	17497.72
5615.611	1	17802.57	5662.872	3	17654.00	5714.548	1	17494.35
5616.225	2	17800.63	5663.426	00	17652.27	5716.005	3	17489.90
5616.772	1*h*	17798.89	5667.358	1	17640.02	5718.715	1	17481.61
5617.469	0a	17796.68	5667.817	0	17638.60	5719.507	2	17479.18
5618.287	1	17794.10	5669.334	1	17633.88	5722.387	1	17470.40
5619.599	0	17789.94	5670.281	1	17630.93	5723.454	6	17467.13
5620.018	1	17788.61	5670.756	1a	17629.46	5724.260	1	17464.67
5620.907	5	17785.83	5670.930	4	17628.90	5724.615	0	17463.59
5622.450	0	17780.91	5672.374	0	17624.43	5725.876	2	17459.76
5622.624	1a	17780.37	5674.085	1	17619.11	5726.590	00	17457.57
5623.081	4	17778.92	5674.506	0	17617.80	5727.052	3	17456.16
5624.306	4	17775.06	5674.984	2	17616.32	5728.552*	10	17451.59
5624.887	0a	17773.21	5676.172	1	17612.63	5729.228	1	17449.52
5625.163	0a	17772.34	5676.605	0	17611.29	5730.314	1	17446.22

*Measured with the interferometer.

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
5731.925	8	17441.32	5787.510	0	17273.81	5849.317*	8	17091.29
5733.254	1a	17437.27	5788.262	4	17271.56	5850.893	2	17086.67
5733.596	1*h*	17436.24	5789.852	2	17266.82	5851.623	2	17084.56
5734.495	1	17433.50	5791.692	0	17261.34	5857.092	1	17068.60
5735.130	6	17431.57	5791.912	4	17260.69	5859.808	5	17060.68
5736.098	1	17428.63	5794.607	2	17252.65	5860.667	0	17058.20
5736.879*	10	17426.25	5794.831	0a	17251.99	5861.179	1	17056.70
5738.415	2	17421.59	5795.982	1	17248.55	5861.590	1	17055.50
5740.089	3	17416.51	5796.594	1	17246.74	5864.462	4	17047.15
5740.519	1	17415.21	5797.222	0	17244.91	5866.666	1	17040.75
5741.108	0	17413.42	5798.959	2	17239.70	5869.262	4	17033.21
5741.405	1a	17412.52	5800.198	0	17236.02	5869.794	1a	17031.67
5741.835	5	17411.21	5801.151	2*h*	17233.19	5870.066	1	17030.88
5743.061	1	17407.51	5801.890	1a*h*	17230.99	5870.304	1	17030.19
5743.918	2	17404.90	5802.479	0	17229.24	5870.567	0	17029.42
5745.374	1	17400.48	5803.932	1	17224.94	5871.623	2	17026.36
5746.324	1	17397.61	5804.677	2	17222.72	5871.952	6	17025.41
5749.015	2	17389.47	5806.099*	8	17218.50	5872.554	1	17023.67
5749.783	1	17387.14	5807.538	0	17214.23	5874.886	0a	17016.90
5751.135	1	17383.06	5807.602	U		5876.063	1	17013.50
5751.545	1	17381.82	5807.680	0	17213.82	5878.102	1	17007.59
5752.350	0	17379.39	5808.289	1	17212.01	5878.496	10	17006.43
5752.860	1	17377.85	5811.498	4	17202.51	5879.201	3a	17004.40
5753.556	1	17375.75	5812.282	1	17200.19	5880.242	0a	17001.40
5755.687	5	17369.32	5812.587*	10	17199.28	5881.998	1a	16996.32
5756.540	1	17366.74	5814.943	6	17192.31	5883.385	0	16992.32
5757.350	5	17364.30	5815.234	1a	17191.46	5883.942	6	16990.71
5757.979	0	17362.40	5816.438	4	17187.90	5884.632	8	16988.72
5759.559	6	17357.64	5817.082	2	17186.00	5885.540	1a	16986.10
5760.392	6	17355.14	5817.600	3	17184.46	5887.346	1	16980.89
5762.719	3	17347.97	5819.279	3	17179.50	5888.167*	10	16978.52
5764.028	1	17344.17	5822.073	4	17171.27	5888.630	0	16977.18
5764.482	1	17342.83	5822.763*	10	17169.22	5889.033	5	16976.03
5764.988	1	17341.26	5824.560	2	17163.93	5891.337	1	16969.39
5765.892	2	17338.58	5825.130	0a	17162.25	5893.683	1	16962.63
5766.284	3	17337.39	5825.569	1	17160.95	5894.011	0	16961.69
5766.989	2	17335.25	5831.016	4	17144.93	5894.164	0*h*	16961.25
5768.241	0	17331.51	5831.488	0	17143.53	5897.735	0a	16950.98
5770.154	1	17325.77	5832.773	5	17139.77	5899.836	0	16944.94
5772.435	1	17318.92	5833.065	4*h*	17138.90	5900.491	1	16943.05
5772.733	0	17318.03	5835.829	3a	17130.78	5901.211	1	16940.99
5773.224	5	17316.56	5836.133	8	17129.89	5903.502	1	16934.41
5774.580	3	17312.48	5836.787	1	17127.97	5904.991	0	16930.14
5775.050*	9	17311.08	5837.077	0	17127.12	5908.315	0	16920.63
5778.984	4	17299.30	5837.409	1a	17126.15	5909.070	0	16918.46
5780.045	2*h*	17296.19	5837.777	2	17125.07	5909.395	3	16917.50
5780.269	0	17295.44	5838.590	0a	17122.66	5909.730	0	16916.57
5780.710	1	17294.13	5839.064	0a	17121.30	5910.165	3	16915.33
5783.004	2	17287.27	5839.918	1	17118.79	5910.808	0	16913.48
5784.400	1	17282.98	5841.453	1	17114.29	5911.750	1*h*	16910.79
5785.208	3	17280.68	5842.574	2	17111.01	5914.940	1	16901.67
5785.768*	7	17279.01	5843.461	0	17108.41	5915.610	0	16899.76
5786.670	1	17276.32	5847.281	0	17097.23	5916.056	5	16898.48

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
5916.506	5	16897.19	5987.639	0	16696.46	6066.631	7	16479.06
5917.190	0	16895.24	5989.239*	5	16692.00	6067.720	7	16476.10
5917.400	0	16894.64	5990.530	4	16688.40	6068.302	0	16474.52
5918.078	5	16892.70	5991.793	1	16684.88	6069.065	00a	16472.46
5919.537	0	16888.55	5992.154	0*h*	16683.88	6069.994	8	16469.93
5920.489	5	16885.83	5994.062*	10	16678.57	6070.482	00	16468.61
5920.799	4	16884.94	5997.801	0	16668.17	6071.608	2	16465.55
5922.004	1	16881.52	5998.105	1	16667.33	6072.003	0	16464.49
5924.830*	9	16873.46	6002.816*	9	16654.24	6074.386	5	16458.03
5925.795	2	16870.70	6004.925	2	16648.40	6078.900	3	16445.80
5928.151	1	16864.10	6006.843	1	16643.08	6080.778*	10	16440.72
5928.363	1	16863.40	6007.182	2	16642.14	6084.100	0	16431.75
5929.509	0a	16860.14	6008.804	0	16637.65	6084.242	0	16431.36
5930.647	1*h*	16856.91	6010.309	0	16633.48	6084.445	0	16430.81
5931.368*	10	16854.86	6010.475	0	16633.03	6084.710	0	16430.10
5932.019	1	16853.01	6011.396	3	16630.47	6085.203	0	16428.77
5932.295	3	16852.22	6012.122	00	16628.47	6085.520	00	16427.90
5936.027	4	16841.63	6012.546	1	16627.29	6085.935	00	16426.79
5937.988	1	16836.04	6014.850	1	16620.92	6086.330	0	16425.72
5938.620*	10	16834.27	6016.640	0	16615.98	6086.858	00	16424.30
5939.432	0	16831.97	6017.350	0	16614.02	6089.807	2	16416.35
5941.139	2	16827.14	6018.291*	10	16611.43	6090.370	0	16414.83
5941.546	1	16825.98	6019.352	0	16608.50	6090.910	7	16413.37
5941.977	3	16824.76	6020.296	1	16605.89	6092.116	0	16410.14
5942.250	0	16823.99	6021.273	9	16603.20	6093.824	0a	16405.53
5942.952	2	16822.00	6021.532	0	16602.48	6095.956*	10	16399.79
5943.554	2	16820.29	6022.291	1	16600.39	6098.218	9	16393.70
5947.302	6	16809.70	6023.751*	10	16596.37	6100.220	1	16388.32
5947.993	1	16807.74	6025.774	1	16590.80	6107.833	1	16367.90
5949.895*	10	16802.37	6026.305	0	16589.34	6108.934	00	16364.95
5955.208	0	16787.38	6027.977*	10	16584.72	6109.479	1	16363.49
5956.210	1	16784.56	6030.048	2	16579.03	6110.778	00	16360.00
5956.502	2	16783.74	6031.474	5	16575.12	6112.114	0*h*	16356.44
5957.216	1a	16781.72	6031.900*	10	16573.94	6112.785	1	16354.65
5959.615	4	16774.97	6032.157	2	16573.23	6113.633	0	16352.37
5959.806	6	16774.42	6032.487	0	16572.33	6116.715	1	16344.13
5960.407	1	16772.74	6035.047	0	16565.30	6117.778	0	16341.29
5963.473	4	16764.12	6041.027	3	16548.90	6118.135	0	16340.34
5967.273	4	16753.44	6042.705	2	16544.31	6119.005	1	16338.02
5967.542	0a	16752.69	6044.344	00	16539.82	6121.213	1	16332.11
5969.317	1	16747.70	6044.651	2	16538.98	6121.787*	10	16330.59
5969.689	2	16746.56	6045.415	1	16536.89	6122.570	0	16328.50
5969.864	1	16746.17	6046.884	0a	16532.87	6124.374	0	16323.68
5970.113	0	16745.48	6047.850	5	16530.23	6126.829	0a	16317.15
5970.310	5	16744.92	6052.369	6	16517.89	6127.242	6	16316.05
5970.933	3	16743.17	6053.265	5	16515.45	6127.640	4	16314.99
5974.139	4	16734.20	6054.724	1	16511.46	6129.948	0a	16308.84
5975.437*	10	16730.55	6056.102	2	16507.71	6130.355	0	16307.76
5978.035	1	16723.28	6057.329	0	16504.37	6130.641	00a	16307.00
5979.114	2	16720.26	6058.319	00	16501.67	6131.832	1	16303.84
5980.866	0	16715.37	6062.618	1	16489.97	6134.313	5	16297.25
5982.561*	6	16710.63	6063.285*	8	16488.15	6135.145	3	16295.02
5983.380	0	16708.34	6064.810	1	16484.00	6135.395*	10	16294.37

*Measured with the interferometer.

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
6137.487	0	16288.82	6274.486	0a	15933.16	6399.475*	10	15621.96
6137.908	1	16287.69	6274.841	3	15932.26	6404.011	1	15610.91
6139.365	2	16283.83	6274.967	2	15931.94	6405.240	0	15607.91
6140.974	0	16279.56	6276.596	1	15927.80	6405.777	0	15606.61
6141.788	1	16277.40	6277.103	3*h*	15926.52	6411.783	2	15591.98
6144.068	2	16271.37	6282.290	00	15913.37	6417.686	0	15577.64
6146.192	3	16265.75	6283.316	0	15910.77	6418.335	00a	15576.06
6146.900	1	16263.87	6285.388*	10	15905.53	6421.913	00	15567.39
6148.119	00	16260.65	6289.715	1	15894.58	6428.113*	10	15552.37
6148.539	1	16259.53	6296.627	0	15877.13	6429.314	3	15549.47
6151.470	4	16251.79	6298.586	1	15872.20	6433.490	3	15539.38
6153.184	2	16247.26	6299.420*	10	15870.10	6434.840	2	15536.11
6155.628	5	16240.81	6300.590	0	15867.14	6437.822	3	15528.92
6159.575	2	16230.41	6302.295	1*h*	15862.86	6440.648	0	15522.10
6161.605*	8	16225.06	6303.485*	5	15859.86	6441.502*	3	15520.05
6162.495	2	16222.72	6306.106	0	15853.27	6444.031	0	15513.96
6164.179	1*h*	16218.29	6316.380	1	15827.48	6445.340	00	15510.81
6165.500	0a	16214.81	6318.641	1	15821.82	6452.061	0	15494.65
6167.736	4	16208.92	6320.379	3	15817.47	6454.985	0a	15487.63
6169.637*	7	16203.93	6323.789	0*h*	15808.94	6456.116	1	15484.92
6174.089	6	16192.25	6326.277	0a	15802.72	6472.189	0	15446.46
6174.888	3	16190.17	6227.063*	10	15800.76	6473.615	2	15443.06
6175.900	1	16187.50	6329.352	2	15795.05	6475.441	0	15438.70
6176.234	3	16186.63	6329.670	1a	15794.25	6495.768	0	15390.39
6177.534	0*h*	16183.22	6329.814	5	15793.89	6499.816	0	15380.80
6182.989*	10	16168.95	6332.486*	5	15787.23	6502.065	1	15375.49
6193.800	0	16140.72	6335.518	0	15779.68	6509.455	1	15358.03
6197.113	5	16132.09	6337.041	0	15775.88	6512.230	0	15351.49
6197.736	2	16130.47	6340.104	0	15768.26	6517.695*	3	15338.61
6199.387*	10	16126.19	6340.574*	7	15767.10	6520.949	1	15330.95
6201.178*	8	16121.52	6345.476	1	15754.91	6527.355*	3	15315.91
6207.309	1	16105.59	6346.962	1*h*	15751.22	6527.820	0a*h*	15314.83
6209.803	0	16099.13	6351.290	1	15740.49	6532.640	1	15303.52
6216.524	0	16081.72	6356.148	0	15728.46	6538.040	0	15390.88
6219.360	0	16074.39	6356.774	0	15726.90	6540.510	2	15385.11
6221.751	2	16068.21	6357.020	0	15726.30	6554.036	4*h*	15353.57
6224.809*	10	16060.31	6358.015	0	15723.84	6559.131	1	15341.72
6230.256*	7	16046.28	6362.483*	6	15712.79	6561.064	2	15337.22
6233.014	5	16039.18	6364.617	1	15707.53	6561.736	1*h*	15235.67
6235.702	1a	16032.26	6365.008	1	15706.57	6562.792	Hα	
6235.887	2a	16031.77	6370.125	1	15693.95	6567.047	1	15223.35
6237.450	0	16027.77	6372.209*	5	15688.81	6569.946	0a	15216.62
6238.388*	10	16025.36	6376.079	1	15679.29	6571.507	2a	15213.01
6241.193	1	16018.16	6377.410	1	15676.02	6571.666	1*h*	15212.66
6246.561	0	16004.39	6379.213	2	15671.59	6572.044	3a	15211.77
6249.161	2	15997.73	6380.110*	4	15669.39	6573.970	0	15207.32
6255.147	0	15982.42	6380.226	0	15669.10	6579.297	00	15195.00
6257.605	0	15976.14	6381.816	0	15665.20	6593.268	0	15162.81
6257.692	0	15975.92	6382.901	2	15662.53	6595.782	1*h*	15157.02
6262.512	2	15963.63	6387.901	1	15650.27	6596.899	1	15154.46
6267.966	3	15949.72	6391.078	1	15642.48	6597.782	1*h*	15152.44
6270.536	3	15943.20	6396.557	00	15629.10	6599.711	0	15148.00
6271.313*	5	15941.22	6397.438	1	15626.94	6602.050	2	15142.64

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
6614.655	1a	15113.77	6820.331	0	14658.00	7040.69	1	14199.2
6617.521	0	15107.23	6821.000	0	14656.57	7044.88	00	14190.8
6619.768	1	15102.10	6825.747	2	14646.38	7046.02	1	14188.5
6622.590	4	15095.67	6830.771	00	14635.60	7049.60	3	14181.3
6623.831	2	15092.84	6832.136	0*h*	14632.68	7053.17	0	14174.1
6629.000	0d	15081.07	6840.886	3	14613.97	7061.23	1	14157.9
6630.391	0	15077.91	6844.094	00	14607.11	7063.02	2	14154.4
6630.998	0	15076.53	6851.659	0c	14590.99	7064.28	00	14151.8
6633.856	1	15070.03	6852.369	00	14589.48	7090.63	0	14099.2
6636.312	0	15064.45	6853.383	1	14587.32	7091.61	0	14097.3
6636.752	0*h*	15063.45	6875.272	3a	14540.88	7095.10	4	14090.4
6643.258	00	15048.71	6876.959	1	14537.31	7095.56	1	14089.4
6647.394	2	15039.34	6896.620	2	14495.86	7097.42	0	14085.7
6661.387	0	15007.75	6898.833	1	14491.21	7099.42	1	14081.8
6666.303	1	14996.70	6899.299	2	14490.24	7104.27	1	14072.2
6675.400	00	14976.25	6899.851	1	14489.07	7105.84	0	14069.1
6677.948	2	14970.53	6900.248	00	14488.24	7106.98	00	14066.8
6685.508	0	14953.60	6902.346	0	14483.84	7108.05	0	14064.7
6694.855	1	14932.73	6902.680	0a	14483.14	7112.41	1	14056.1
6696.755	3	14928.49	6903.028	0	14482.41	7112.65	3	14055.6
6699.653	0	14922.03	6909.215	1a	14469.41	7114.25	1	14052.4
6701.861	1a	14917.11	6909.322	0	14469.22	7116.70	1	14047.6
6707.600	1	14904.35	6909.672	1a	14468.48	7121.44	1	14038.2
6716.255	1a	14885.15	6911.880	00	14463.86	7126.81	0	14027.7
6733.505	0a	14847.02	6915.845	00	14455.57	7133.39	1	14014.7
6734.727	00	14844.32	6916.463	00a	14454.28	7138.14	2	14005.4
6735.532	0a	14842.77	6920.321	00	14446.22	7139.32	1	14003.1
6743.556	1	14824.89	6921.508	0	14443.74	7152.26	0a	13977.7
6748.874	0	14813.20	6929.229	1	14427.64	7153.47	0a	13975.4
6756.262	3	14797.01	6935.913	1	14413.75	7155.71	2	13971.0
6757.374	0	14794.57	6938.826	0	14407.70	7159.03	0	13964.5
6759.138	00	14790.72	6938.950	1	14407.43	7160.30	1a	13962.1
6768.446	0a	14770.37	6940.421	4	14404.38	7166.48	00	13950.0
6770.090	0b	14766.78	6949.057	00b	14386.48	7168.81	8	13945.5
6772.459	00	14761.62	6955.351	0	14373.46	7170.04	0	13943.1
6780.784	0a*h*	14743.50	6961.295	1	14361.19	7172.71	1	13937.9
6781.880	00	14741.11	6962.320	2	14359.08	7173.07	2	13937.2
6783.264	00	14738.10	6973.300	00c	14336.47	7176.28	5	13931.0
6789.674	2	14724.20	6977.893	2	14327.04	7179.37	1a	13925.0
6794.083	3	14714.64	6987.598	00	14307.13	7179.52	3a	13924.7
6796.339	00	14709.75	6988.708	0	14304.86	7182.90	1	13918.1
6797.377	00	14707.50	6990.606	1	14300.97	7184.04	6	13915.9
6800.479	00	14700.79	7001.54	0	14278.6	7184.37	2	13915.3
6802.637	0	14696.13	7004.25	00	14273.1	7188.53	1	13907.2
6805.400	1a*h*	14690.17	7010.78	0a*h*	14259.8	7194.88	1	13895.0
6806.438	4	14687.93	7014.87	0	14251.5	7195.66	9	13893.4
6809.190	00	14681.98	7022.34	2a	14236.3	7200.49	0a	13884.1
6815.253	0*h*	14668.92	7023.85	0	14233.3	7201.37	1	13882.4
6815.809	0*h*	14667.73	7030.67	00	14219.5	7203.30	1	13878.7
6817.480	1	14664.14	7031.42	00	14218.0	7205.44	1c	13874.6
6817.670	1a	14663.73	7035.12	0	14210.5	7209.95	2a	13865.9
6818.425	1	14662.10	7035.61	2	14209.5	7210.22	3	13865.4
6818.587	0*h*	14661.75	7039.16	0b	14202.3	7212.13	0	13861.7

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
7214.88	1	13856.4	7319.55	1a	13658.3	7504.77	1	13321.2
7217.28	00	13851.8	7328.12	4	13642.3	7506.95	3	13317.3
7218.21	1	13850.0	7333.46	2	13632.4	7507.84	1a	13315.7
7222.65	0	13841.5	7344.64	0	13611.6	7512.27	1b	13307.9
7223.11	00b	13840.6	7350.00	4	13601.7	7520.29	00	13293.7
7227.88	2a	13831.5	7350.70	4	13600.4	7521.12	00	13292.2
7230.66	4	13826.2	7353.73	00	13594.8	7522.17	00a	13290.4
7231.06	4	13825.4	7354.00	1	13594.3	7524.64	9	13286.0
7231.19	1	13825.2	7355.87	0	13590.8	7531.72	00	13273.5
7240.13	1	13808.1	7360.33	1	13582.6	7534.85	0	13268.0
7240.57	6	13807.3	7361.33	0	13580.8	7536.90	0	13264.4
7241.50	0c	13805.5	7365.33	1	13573.4	7537.24	1	13263.8
7243.51	1a	13801.7	7366.62	0a	13571.0	7538.32	3	13261.9
7244.11	3	13800.5	7368.06	0a	13568.4	7541.95	2a	13255.5
7244.72	2	13799.4	7368.88	00a	13566.9	7544.99	3	13250.2
7250.89	2a	13787.6	7369.93	00	13564.9	7558.63	00	13226.3
7253.28	10	13783.1	7371.62	2	13561.8	7559.07	0	13225.5
7254.02	3	13781.6	7374.87	3	13555.8	7559.60	0	13224.6
7259.04	00	13772.1	7378.72	0	13548.8	7561.29	1	13221.6
7259.88	1	13770.5	7384.42	1	13538.3	7565.13	00	13214.9
7261.62	1	13767.2	7395.04	3	13518.9	7571.59	1a	13203.6
7262.13	0	13766.3	7396.07	2	13517.0	7576.41	1	13195.2
7262.32	1a	13765.9	7396.97	0	13515.3	7579.42	0a	13190.0
7263.58	1	13763.5	7398.18	1	13513.1	7584.40	0a	13181.3
7266.63	2	13757.8	7404.15	00	13502.2	7585.01	1	13180.3
7267.15	2a	13756.8	7406.03	1a	13498.8	7589.82	0	13171.9
7269.06	0b	13753.2	7414.00	1	13484.3	7593.61	00	13165.3
7269.96	7	13751.3	7414.45	0	13483.5	7593.94	00a	13164.8
7275.53	1	13740.9	7421.67	1	13470.4	7595.05	00	13162.9
7279.71	1	13733.0	7422.82	0a	13468.3	7595.85	0a	13161.5
7280.06	2a	13732.4	7428.39	0	13458.2	7597.06	3	13159.4
7283.90	1	13725.1	7430.38	0	13454.6	7597.27	0	13159.0
7285.60	0	13721.9	7431.41	1	13452.7	7598.02	1	13157.7
7286.32	1	13720.6	7434.09	2a	13448.8	7602.53	1a	13149.9
7287.50	2	13718.4	7435.05	1	13446.1	7603.43	4	13148.3
7288.94	3	13715.7	7438.70	00	13439.5	7606.36	4	13143.3
7291.55	1	13710.7	7443.85	00	13430.2	7613.89	0	13130.2
7295.42	3	13703.5	7447.66	00	13423.3	7620.89	1a	13118.2
7300.05	0b	13694.8	7449.14	4	13420.7	7622.67	1	13115.2
7300.28	0a	13694.3	7449.39	1	13420.2	7623.53	00	13113.7
7301.48	1	13692.1	7455.70	1	13408.9	7641.76	1	13082.4
7302.16	0	13690.8	7459.45	4	13402.1	7645.03	0	13076.8
7303.62	1b	13688.1	7463.53	0	13394.8	7647.52	1	13072.5
7306.72	0c	13682.3	7464.61	00	13392.9	7647.85	0	13072.0
7306.94	1	13681.9	7466.20	0	13390.0	7650.75	3	13067.0
7307.69	1	13680.4	7468.20	0	13386.4	7651.82	00a	13066.5
7309.56	5	13677.0	7469.17	1	13384.7	7654.47	1	13060.7
7310.28	2	13675.6	7474.51	2	13375.1	7660.28	1a	13050.8
7311.09	0	13674.1	7478.42	1	13368.2	7660.77	0a	13049.9
7312.82	2	13670.9	7487.67	00	13351.6	7661.46	2	13048.8
7313.56	1	13669.5	7488.22	1	13350.6	7665.32	00	13042.2
7316.86	0a	13663.3	7489.45	1	13348.4	7672.43	00	13030.1
7318.02	1	13661.1	7497.88	00	13333.4	7679.79	1	13017.6

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
7685.53	2	13007.9	7933.14	2a	12601.9	8133.64	0	12291.3
7690.36	1a	12999.7	7937.31	1	12595.3	8139.20	0	12282.8
7692.09	1a	12996.8	7941.39	00	12588.8	8157.02	00	12256.0
7692.71	1	12995.7	7942.68	1b	12586.8	8159.20	00	12252.7
7694.79	0	12992.2	7950.35	0	12574.6	8159.56	00	12252.2
7699.46	00	12984.4	7953.44	1a	12569.7	8164.64	8	12244.6
7702.58	00a	12979.1	7957.03	0b	12564.0	8187.75	0	12210.0
7704.99	1a	12975.0	7962.50	0b	12555.4	8190.93	0	12205.3
7706.28	0	12972.9	7963.40	0b	12554.0	8193.71	00	12201.1
7706.66	00	12972.2	7967.37	1a	12547.7	8201.55	0	12189.5
7717.79	00a	12953.5	7968.73	0	12545.6	8202.58	00	12187.9
7719.09	1a	12951.3	7970.14	2b	12543.4	8203.43	00	12186.7
7721.70	1	12947.0	7980.02	00	12527.9	8206.75	00	12181.7
7723.01	0	12944.8	7981.20	0	12526.0	8208.42	00a	12179.3
7724.38	0	12942.5	7984.22	0	12521.3	8222.90	2	12157.8
7726.36	00	12939.1	7985.86	1	12518.7	8235.72	1	12138.9
7732.74	2	12928.5	7987.50	00	12516.1	8237.74	1*h*	12135.9
7733.28	1a	12927.6	7991.58	1	12509.7	8238.68	1	12134.5
7740.01	1	12916.3	7993.58	0b	12506.6	8240.75	3	12131.5
7743.20	00	12911.0	7995.05	00	12504.3	8242.93	00	12128.3
7744.79	1	12908.4	7997.20	2	12500.9	8246.83	1	12122.5
7752.07	00	12896.2	7998.12	00	12499.5	8254.00	0	12112.0
7757.16	00	12887.8	8000.75	00	12495.4	8254.98	0	12110.6
7768.08	0	12869.7	8006.92	1	12485.8	8255.80	1	12109.4
7774.20	00b	12859.5	8007.89	00	12484.3	8265.26	00	12095.5
7789.78	4	12833.8	8013.32	1	12475.8	8269.37	00	12089.5
7796.70	0	12822.4	8018.71	1	12467.4	8270.63	00	12087.7
7798.82	0	12818.9	8019.35	00	12466.4	8273.26	8	12083.8
7800.40	00	12816.3	8023.09	00b	12460.6	8287.05	00	12063.7
7802.59	0	12812.7	8029.35	00	12450.9	8303.65	0	12039.6
7803.95	0	12810.5	8036.57	1	12439.7	8317.42	1	12019.7
7804.30	1a	12809.9	8042.14	0b	12431.1	8319.63	00a	12016.5
7805.71	0	12807.6	8047.28	0	12423.1	8330.42	7	12000.9
7807.21	0	12805.2	8054.55	2	12411.9	8334.33	0	11995.3
7812.42	3	12796.6	8059.76	00	12403.9	8335.30	00	11993.9
7812.97	1	12795.7	8071.20	1	12386.3	8344.05	0	11981.3
7829.62	00	12768.5	8085.02	1	12365.2	8349.52	10	11973.5
7834.17	0	12761.1	8086.36	0	12363.1	8351.03	00a	11971.3
7836.93	0	12756.6	8091.60	0	12355.1	8357.12	0	11962.6
7838.14	0	12754.6	8092.77	00	12353.3	8359.73	0	11958.8
7856.61	1	12724.6	8094.74	0	12350.3	8366.89	3	11948.6
7863.50	00	12713.5	8100.14	1a	12342.1	8370.06	1a	11944.1
7875.43	1	12694.2	8104.22	1	12335.9	8371.76	00c	11941.7
7885.27	00	12678.4	8115.02	1c	12319.5	8372.06	1c	11941.2
7886.90	0	12675.8	8116.42	00	12317.3	8374.11	00	11938.3
7896.00	00	12661.2	8117.30	00	12316.0	8381.16	2	11928.2
7900.24	1	12654.4	8119.36	00	12312.9	8387.17	1	11919.7
7902.61	0a	12650.6	8123.41	00	12306.7	8398.26	3	11904.0
7903.52	0a	12649.1	8124.69	00	12304.8	8399.66	00	11902.0
7905.25	1	12646.3	8126.03	0b	12302.8	8406.62	0a	11892.1
7923.44	1	12617.3	8129.06	1	12298.2	8413.54	0	11882.3
7926.48	00	12612.5	8130.40	0	12296.1	8414.40	0a	11881.1
7926.72	00	12612.1	8130.77	3	12295.6	8422.07	0	11870.3

TABLE I—*Continued*

Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)	Wave-Length	Intensity	Wave-Number (*in Vacuo*)
8424.54	00	11866.8	8522.23	1c	11730.8	8663.91	3a	11539.0
8443.57	2a	11840.1	8522.89	1	11729.9	8670.81	1c	11529.8
8444.62	2	11838.6	8524.27	1	11728.0	8715.12	0c	11471.2
8447.72	00d	11834.3	8528.11	2	11722.7	8724.18	2c	11459.3
8463.56	0	11812.1	8535.66	00	11712.3	8747.00	0b	11429.4
8470.05	0	11803.1	8541.74	00	11704.0	8794.93	1a	11367.1
8478.71	0c	11791.0	8546.27	6	11697.8	8807.77	1a	11350.5
8486.08	2	11780.8	8559.94	00b	11679.1	8875.44	0	11264.0
8496.20	0c	11766.7	8561.66	00a	11676.8	8876.30	0b	11262.9
8509.33	00	11748.6	8574.92	0b	11658.7	8884.57	0c	11252.4
8512.45	00	11744.3	8584.03	00b	11646.3	8896.82	0b	11236.9
8516.92	0b	11738.1	8585.29	00a	11644.6	8898.90	0c	11234.3
8520.37	2	11733.4	8629.12	00b	11585.5	8902.01	0b	11230.3
8520.71	00	11732.9	8657.19	1a	11547.9			

THE WIDTH OF THE D ABSORPTION LINES IN SODIUM VAPOR

By ARTHUR S. FAIRLEY

ABSTRACT

Approximate measurements of the width of the D lines in absorption in sodium vapor under controlled conditions of temperature were made, over the range 180°–380° C., with a column of vapor 21 cm long. Where the measured width was greater than 0.25 A the results roughly confirm J. Q. Stewart's suggestion that the widening is due to a cause analogous to "broad tuning"; since the observed widths fit the formula he has derived from the classical theory of a "*simple linear oscillator.*"

Direct comparison between the artificial spectrum and the solar spectrum indicated that the solar (D) lines may be produced by a column of only about 4×10^{13} *non-ionized sodium atoms per square centimeter in the solar atmosphere.*

DESCRIPTION OF APPARATUS

The apparatus used consisted of a source of continuous white light free from the absorption lines of sodium, a suitable container for the sodium, and a spectroscope of sufficient resolving power, with a micrometer eyepiece for measuring the width of the lines.

The source of light used was a 500-watt projection bulb, with a reflector and collimating lenses. The container for the sodium furnished some difficulties, as the ends had to be transparent and reasonably flat, and the material such as would stand a fairly high temperature. Pyrex tubing was used, with ends made on it by a device developed for the purpose.

This device included a flat plate of steel, which was polished and sputtered with platinum; upon this rested a heavy iron ring, having a thickness of about $1\frac{1}{2}$ inches and an inside diameter about $\frac{1}{16}$ inch larger than the glass tubing used, which was of $\frac{1}{2}$-inch inside diameter and medium thickness. A plunger, of the same diameter as the inside of the glass, and having a flat and polished end, was arranged to move up and down by means of an old lathe chuck mounted vertically. A piece of pyrex tubing about 2 inches long was heated at one end until it sealed itself. It was then placed in the iron ring, with the closed end down and resting on the polished steel plate. Then the plunger was brought down so that it entered the tube and pressed the soft part of the glass into a thin sheet between itself

and the steel plate. The plunger, ring, and plate had first been heated by means of Bunsen burners. After the operation the glass was removed and annealed in a flame for a few minutes. The quality of the ends thus produced on the tubes was somewhat better than that of ordinary window glass, but inferior to plate glass.

To make a container for the sodium, one of these ends was sealed to either end of another piece of pyrex tubing, to which had previously been attached a bulb containing metallic sodium, and a smaller tube for evacuating. After a finished tube had been evacuated for some time on a mercury diffusion pump, the sodium was distilled from the bulb into the tube, and the bulb sealed off. The tube was allowed to evacuate for some time longer, and then was sealed off and was ready for use. Care was taken to have the sodium deposited over all of the inside of the tube except the ends.

The spectroscope used had a Rowland plane grating of 14,483 lines to the inch, collimator, and a view telescope of focal length $1\frac{1}{2}$ m. and 3-inch aperture. It had a resolving power of about 90,000. In front of half the slit was placed a total reflecting prism, so that a comparison spectrum of the sun could be viewed at the same time as that of the sodium. The micrometer eyepiece could be removed, and a plate-holder substituted.

PROCEDURE

In using the apparatus the pyrex tube containing the sodium was placed in an ordinary cylindrical electric furnace, having ends of thick asbestos, in each of which a window of plane pyrex glass an inch in diameter had been inserted. (In the earlier measures the sodium tube was heated in an oil bath, both paraffin and Russian oil being tried. Neither of these, however, could be heated to the required temperature without becoming dark brown and practically opaque. In the later measures the furnace was used.) Before being inserted in the furnace the tube was wrapped with sheet copper, to insure an even distribution of heat throughout its length. The furnace was about twice as long as the tube, for the same reason. The tube was held in the center of the furnace by means of projecting loops of copper wire. A mercury thermometer was inserted

through the end of the furnace, so that its bulb came in contact with the outside of the tube at the center.

In making a run, the usual procedure was to put a current of 4 or 5 amp through the furnace. This caused it to heat up fairly slowly, without much lag in the temperature of the tube. The temperature was noted when the D lines first appeared, in absorption. Measurements were made of the widths of the lines as soon as they were wide enough to measure with any degree of accuracy. This was when they were approximately 0.09 A wide. The measurements were continued until the lines became so broad that it was impossible to see enough light between them to tell where to set the cross-hairs of the micrometer. The current was then decreased in the furnace, so that the temperature began to fall, and readings of the widths were taken as the lines narrowed. The temperature at which they disappeared was noted. This was somewhat lower than that at which they appeared, probably because they could be seen when they were fainter after one knew exactly where to look for them.

In making a reading of the width of a line when the tube was being heated, the temperature was run up about 2° higher than that desired, and the current was shut off while the measurement was being taken. At the end of the measurement the temperature was about 2° lower than that desired. This was done to correct for any lag that there may have been between the temperature of the tube and that of the furnace. It was assumed that the temperature of the tube was at the desired point during the reading. In making the measurements with the temperature decreasing the opposite practice was followed; that is, the current was on while the reading was being taken and off otherwise.

The measurements were all made with the micrometer eyepiece of the spectroscope. Two cross-hairs, forming an X of small angle, were movable, by the micrometer screw. The spectrum as seen in the eyepiece ran horizontally, the two D lines, of course, being vertical. The center of the X was set just on one edge of a line, then on the other edge, and the width of the line was thus determined from the micrometer readings. Consistent readings could be obtained by a single observer, although a second observer might

differ systematically in judging the degree of contrast which marked the edge.

Five different sodium tubes were used, each 21 cm long. A tube could not be used very long before the action of the sodium caused the glass to become opaque.

RESULTS

Table I gives specimen results of a sample run; and Table II gives the mean results of all readings; Figure 1 shows these plotted,

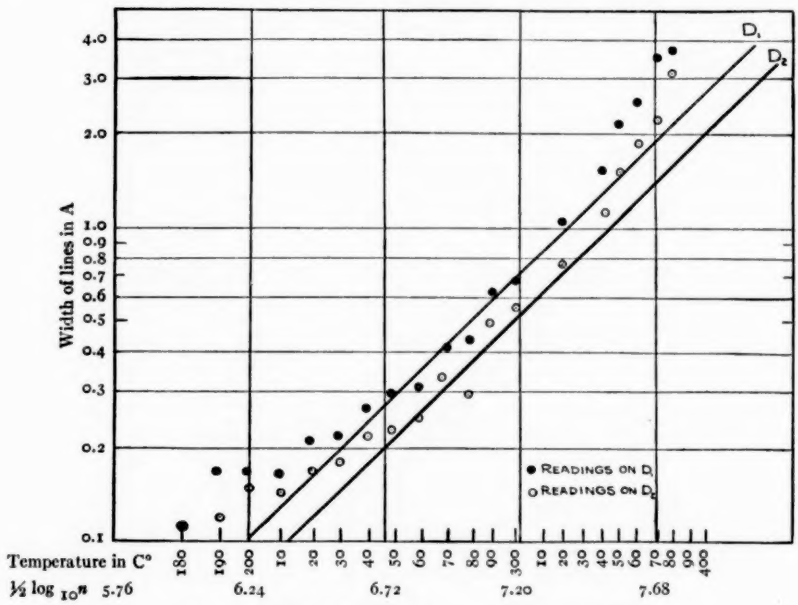

FIG. 1.—Width of D lines at various temperatures

with logarithms of line-widths as ordinates against values of $\frac{1}{2}$ log n as abscissae, where n is the number of molecules of sodium per centimeter of vapor in the tube. Computations to obtain this quantity n were based on the formula of E. Edmondston and A. Egerton[1] for the vapor pressure of sodium at various temperatures (Table III).

[1] *Proceedings of the Royal Society*, A, **113**, 520, 1927.

It was noted that the dark lines expanded equally on either side of their original positions, i.e., the centers did not shift.

TABLE I

SPECIMEN RUN
(Length of Tube, 21 Cm)

T. ° C.	Width of Lines in Angstroms		T. ° C.	Width of Lines in Angstroms	
Temperature Increasing	D_1	D_2	Temperature Decreasing	D_1	D_2
200	0.18	0.15	380	4.80	4.20
220	.24	.18	370	4.08	2.70
240	.27	.21*	350	2.70	1.80
260	.30	.24	330	1.80	1.08
280	.48	.36	310	1.38	0.96
300	0.60	.42	290	0.90	.60
320	1.14	0.84	270	.60	.42
340	1.92	1.20	250	.36	.24
360	3.12	2.28	230	.24	.18
380	4.50	3.60	210	.18	.12
			190	.15	.09
			180	0.12	0.09

* The artificial dark sodium lines were observed as equal in width to the solar D lines, at this point.

TABLE II

MEAN OBSERVED WIDTHS IN ANGSTROMS
(With 21-Cm Tubes)

T. ° C.	D_1	D_2	Number of Observations to Form Mean
180	0.11	0.09	6
190	.17	.12	6
200	.17	.15	8
210	.17	.14	10
220	.21	.17	10
230	.22	.18	10
240	.27	.22	10
250	.30	.23	10
260	.31	.24	10
270	.41	.33	10
280	.44	.29	10
290	.63	.44	10
300	0.68	.48	10
320	1.08	0.76	12
340	1.53	1.13	12
350	2.25	1.50	12
360	2.58	1.86	12
370	3.36	2.25	8
380	3.78	3.12	8

A spectrogram of a pair of wide lines showed the centers apparently perfectly black. The edges passed gradually into the background.

TABLE III

ASSUMED RELATION* BETWEEN THE TEMPERATURE AND THE DENSITY OF SODIUM VAPOR

T. ° C.	n	$\frac{1}{2} \log n$
180	9.4×10^{11}	6.00
190	1.6×10^{12}	6.13
200	2.8×10^{12}	6.23
210	4.7×10^{12}	6.35
220	7.7×10^{12}	6.46
230	1.2×10^{13}	6.56
240	2.0×10^{13}	6.66
250	3.2×10^{13}	6.75
260	4.9×10^{13}	6.85
270	7.4×10^{13}	6.94
280	1.1×10^{14}	7.03
290	1.5×10^{14}	7.10
300	2.2×10^{14}	7.19
310	3.2×10^{14}	7.28
320	4.6×10^{14}	7.35
330	6.4×10^{14}	7.42
340	8.9×10^{14}	7.49
350	1.2×10^{15}	7.55
360	1.6×10^{15}	7.61
370	2.2×10^{15}	7.68
380	2.9×10^{15}	7.74

* This relation is calculated from the formula of Edmondston and Egerton (*loc. cit.*); n = number of sodium molecules per cubic centimeter.

CONCLUSIONS

Referring to Figure 1, it appears that, from a range of 0.25–1.0 A of the width of the line, this width increases as the square root of the number of molecules per unit column in the line of sight—since the straight line with a slope of 45° fits these observations. This result agrees with the equation deduced by J. O. Stewart[1] from classical theory. This is equation (9) of the paper cited,

$$\Delta=(8.15\times10^{-13})\lambda\sqrt{nL/k}\ , \tag{1}$$

where n = number of active molecules per centimeter, L = the length of the absorbing column in centimeters, k = the "contrast factor," Δ = the "observable line-width" in angstroms, λ = wave-

[1] *Astrophysical Journal*, **59**, 30, 1924.

length. The numerical factor is $\sqrt{6\pi a}$, where a is the radius of the electron, which may be taken as 1.88×10^{-13} cm.

The contrast factor, k, is here such that the ratio (brightness in the line)/(brightness of the continuous background) is $1/(1+k)$ at the observed edge, strictly e^{-k}, since all the scattered light is lost through the sides of the long tube; but k is small at the edges. The value of k depends on the sensitivity of the method employed to determine the edge of the line—the observed width, by (1), increasing as $\sqrt{k}$ decreases.

Substituting the proper λ for the sodium lines (5890 A) and taking logarithms to the base 10, equation (1) reduces to

$$\log \Delta \text{ (angstroms)} = \tfrac{1}{2} \log n + \tfrac{1}{2} \log L/k - 8.32\,. \tag{2}$$

Equation (2) assumes that all the sodium molecules are active in producing D_1.

The middle range of the observations of D_1 in Figure 1 fits this equation very approximately, provided $\frac{1}{2}\log L/k$ is assumed as 0.95. (The left-hand straight line with a slope of 45° represents the theoretical curve.) Since L is 21 cm this corresponds to $k=0.26$, and a minimum observed contrast at the edge of the line of $1/(1+0.26)$ or about four units of brightness in the line to five in the background.

Above the width 1.0 A, the line-width was observed to increase more rapidly than the square root of the number of sodium molecules in the line of sight. This also is in accordance with Stewart's theory, for, as indicated in the paper cited, the foregoing equations (1) and (2) do not hold when the vapor density is great enough to make the polarization appreciable (due to reinforcement of the scattered waves by neighboring molecules). These equations break down when the quantity $na\lambda^2 c/\alpha$ is no longer small (where c is the speed of light and α^2 is two-thirds the mean square molecular velocity of thermal agitation). Computations show that in the present work this condition intervenes above about 330° C. (This condition would not intervene in stellar atmospheres; for, owing to the far greater effective L, the n is much smaller for lines of given width; also α is larger.)

The narrower observed lines, on the contrary, were wider than equation (2) predicts. Naturally the measures of narrower lines were the less reliable. The departure here may have been due to imperfect adjustment of the spectroscope, although this was not established. Or again the observer may have estimated the edge of the narrower lines as corresponding to less contrast between edge and background—i.e., to a smaller value of k. A systematic effect of widening might thus be introduced at the smaller width. Obviously further study is necessary.

If half as many molecules are active in producing the line D_2 as D_1 (as generally believed) the ratio of the widths, according to equation (2), should be $\sqrt{1/2}$, which is about that observed.

In comparing the absorption lines produced in this way with the D lines in the solar spectrum, it was noticed that these were quite black at the center, while the solar lines were not. It may be supposed that there is enough thermal emission in the upper layers of the solar atmosphere to account for this difference. Of course, there is none in the tubes.

The amount of ionization of the sodium vapor (ionization potential, 5.13 volts) in the tubes was very small indeed, when calculated by Saha's formula, being, at 380° C., of the order 1 part in 10^{-31}. On the average, there was not one ion in the whole tube. Hence it was impossible that the widening of the lines noted could have resulted from the action of ions or free electrons in the gas. Professor A. S. Eddington[1] has advanced the opinion that the width of stellar spectral lines may be due to the influence of electric fields of neighboring ions in an ionized gas. That theory is inadequate to explain the widening of the D lines under the conditions dealt with in the present study. These measurements (for lines wider than 0.25 A), on the other hand, are compatible with Stewart's idea that the width is an effect which can be dealt with by the equations of classical electromagnetic theory—being, indeed, analogous to "broad tuning" in radio, and ascribable to electromagnetic damping of "radiation resistance." This is an effect of intrinsic width, and the contribution of each atom is theoretically independent of the density of the vapor—provided the density is low. The total number

[1] *The Internal Constitution of Stars*, p. 355, 1926.

of active atoms per square centimeter in a column in the line of sight is the significant parameter.

As has been stated, the measured width of the narrowest lines observed decreased even less rapidly than the square root of the number of molecules per unit column in the line of sight. If mutual influence of neighboring molecules were an important factor in line-width at these low densities, the width would decrease rapidly with the density—contrary to these observations.

The work described above was suggested and supervised by Professor J. Q. Stewart. It was undertaken with the idea that it would be of some value in determining the amount of material necessary to produce a line of given width in the atmosphere of a star.

The results given are, from a quantitative standpoint, only preliminary. In particular the widths at the lower vapor-densities require further study; and also the contour of the wide lines at the higher vapor-densities should be examined (preferably with a microphotometer). The method of measurement used might well be improved upon in future work. The uncertainty in the value of the contrast at the edges (k) should be removed by microphotometric measurements.

Referring to Table I, a matter to be emphasized is that direct comparison with the solar spectrum—the artificial absorption spectrum being obtained beside the Fraunhofer spectrum in the same instrument—showed that at 240° the line-widths were equal. According to Table III, only 2.0×10^{13} sodium molecules were present per centimeter at this temperature, or 4.2×10^{14} molecules per square centimeter in a column in the 21-cm tube. The formula employed for calculating vapor-density seems reasonably trustworthy. Consequently this measurement indicates that the Fraunhofer (D) lines are due to only about 4×10^{14} non-ionized sodium atoms in a column 1 cm square above the photosphere. *This is about 7,000 times less than the number of molecules present in a centimeter of atmospheric air.*

PRINCETON UNIVERSITY OBSERVATORY
July 1927

SOME STRUCTURAL FEATURES OF THE GALACTIC SYSTEM[1]

BY FREDERICK H. SEARES

ABSTRACT

Analysis of systematic deviations from mean stellar distribution.—The greater part of the *observed systematic deviations* have been represented (*Contribution* No. 346) by curves of the form

$$\Delta = a + b \cos (\lambda - L') ,$$

in which the parameters depend on the limiting magnitude ($m=9$, 11, 13.5, 16, and 18) and the galactic latitude ($+70°$ to $-70°$). The *theoretical deviations* for an eccentric location of the sun within a stellar system having spheroidal symmetry, with allowance for error in the position of the galactic pole, have the form

$$\Delta = s \pm G + F \cos (\lambda - L) \mp k \cos (\lambda - L_0) ,$$

the upper signs referring to northern latitudes, the lower to southern. Higher harmonics in the theoretical expression may be neglected. For any limiting magnitude, F and G depend on the latitude, the space density, and ξ and ζ, the co-ordinates of the center of the system relative to the sun. L is the longitude of the center, s a systematic correction to the mean distribution table, L_0 the longitude of the true galactic pole relative to the adopted pole, and k a function of the polar distance of the true pole. Simple *substitutions reduce* the *theoretical* expression *to the observed form* and connect the parameters, F, G, L, etc., with the known quantities a, b, and L'. Values of the *parameters* for the *same limiting magnitude* derived from data for 10° zones of latitude are generally *accordant* (Table II); but the *positions of* both the *pole and* the *center* of the system *depend on* the *magnitude* (Table III). The hypothesis underlying the theoretical formula holds for any given limit, but the characteristics of the spheroid representing the system vary with the magnitude.

Higher harmonics in observed surface densities.—The agreement of the first harmonics derived from different zones of latitude does not exclude the existence of higher harmonics. These are involved in the residuals (Table IV) given by the theoretical formula. The *average deviation* from *spheroidal symmetry*, for any limiting magnitude, averages 25–30 per cent in the numbers of stars for low latitudes and sinks to 8 or 10 per cent for latitudes above 40°. The largest negative residuals are associated with well-known obscured areas.

Dominance of the local cluster.—Combined with existing data, the positions of the center and the pole form a sequence (Fig. 5) which begins with values for the local cluster of bright helium stars and terminates with the center for the globular clusters and the pole of the Milky Way. The sequence indicates that the *bright helium stars* are but the *nucleus* of a much *larger local system*, having a *diameter of 6000 parsecs* or more, which includes stars as faint as the *fifteenth* or *sixteenth magnitude* and is responsible for *three-fourths* of the total *space density near* the *sun* and one-half the mean total density in the galactic plane at distances of about 700 parsecs. The estimated diameter

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 347.

of the local cluster presupposes a similar distribution of spectral types in the local and larger systems.

Attempts to *locate the center* of the stellar system by neglecting the local cluster and supposing that only spheroidal symmetry need be considered lead to contradictory results which strengthen the conclusion that the *cluster cannot be ignored*. The assumptions thus involved would require a large increase in space density toward the center for a thousand parsecs or so, and a very slow decline in the *average* density for all longitudes in the galactic plane, if not an actual decrease preceding the decline. This disagrees with the known behavior of the mean space density and with the evidence of ordinary star counts, which show that *no* general *increase in space density* occurs even in the *direction of* the *center*.

Ratios of numbers of stars and total light.—The dominance of the local cluster prevents the location of the center by the method originally contemplated. The formulae may be used, however, to calculate the *ratios of* the total *numbers of stars* in the directions of the *center* and *anti-center* and of the galactic poles (Q_ξ and Q_ζ [Tables II, III]). These are important observational results, because they lead to the ratios of total light ($B_\xi=3.04$, $B_\zeta=1.29$), which are equal to the *ratios* of the *integrated space densities* in these same directions.

Distance of the sun from galactic plane.—The *distance* of the sun from the galactic *planes of the systems* of stars brighter than the *sixteenth* and *eighteenth magnitudes is o* with an estimated uncertainty of 8 parsecs. The planes considered are those defined by equality in numbers of stars in northern and southern galactic hemispheres between latitudes 30° and 70°. The *distance from* the plane of the *local cluster*, which lies to the south of the larger system, is in *excess of 30 parsecs*, a very uncertain estimate being 40–50 parsecs. Differences in definition, method, and data must be considered in comparing these results with others derived from special classes of objects.

Provisional space densities.—Calculations for the directions of center, anti-center, and poles were made by Schwarzschild's method. The gaussian function for luminosity required by the method assigns frequencies to stars of low luminosity which are much too small. The calculated densities therefore decrease too slowly with increasing distance; but their *relative values* in different directions should be *little affected*. The results, as usual, are smoothed values. The steady decline with increasing distance does not exclude the existence of fluctuations of large amount, such as would be produced by a succession of star clouds scattered over the galactic plane. The *distribution* in the directions of center and anti-center (Fig. 7, Table VII) is *very asymmetrical*. The ratio of the integrated densities satisfies the condition imposed by the ratio of total light, $B_\xi=3.04$.

Resolution of space density; component systems.—The asymmetrical distribution-curve of density in the direction of the center may be resolved into a symmetrical or nearly *symmetrical curve*, concentric with the sun and *representing* the *local cluster*, and an *asymmetrical curve* representing the *larger system*. The analysis is not unique; but a resolution which gives much regularity to the larger system results in characteristics for the local cluster which agree with those indicated by the evidence of Figure 5. A resolution into *two symmetrical systems* is *impossible*.

The larger system.—The calculated *density* of the larger system increases toward the center to a *maximum*, distant a *thousand parsecs or more*, according to the degree of asymmetry admitted in the local cluster. This maximum was formerly identified with the center of the larger system; but the contours of equal density, although not

concentric, agree in indicating that the *geometrical center* is at a much *greater distance*. The *asymmetry in* the *density* distribution of the larger system and the apparent decrease in density beyond the neighboring maximum, before the geometrical center is reached, present difficulties which find a probable *explanation* in the *influence of obscuration analogous* to that seen in highly inclined or edge-on *spiral nebulae*. This leaves the *distance of the center* undetermined, but opens the way to a provisional acceptance of Shapley's estimate of *20,000 parsecs*, derived from the globular clusters. The agreement of the longitude of the center for the faintest limiting magnitude ($m=18.0$, $L=319°$) with Shapley's value for the clusters ($L=325°$) is excellent.

Analogy between stellar systems and spiral nebulae.—The analysis leads to an *amplified hypothesis* which regards the *galactic system* as a vast organization from 60,000 to 90,000 parsecs in diameter, *resembling* the highly resolved *spiral nebulae*. The *local cluster* is an aggregation of *many million stars*, perhaps corresponding to one of the knots or condensations in the arms of a spiral, although apparently of rather exceptional size relative to the larger system. All the *known peculiarities* of stellar *distribution* are well *explained*, qualitatively at least, *by this concept of structural relations*.

I. INTRODUCTION—THE PROBLEM OF SPACE DENSITY

A direct determination of the space density of stars by measuring the parallaxes of a representative collection of objects is so restricted in its possibilities that recourse is usually had to statistical methods based on the observed distribution over the face of the sky. The most important observational item for the statistical solution of the problem consists of surface densities, expressed usually by values of log N_m, where N_m is the number of stars per square degree brighter than m. A second essential is the frequency distribution of stellar luminosities. This may, and probably does, vary from point to point in space. A good approximation for the average frequencies of stars brighter than the seventh or eighth absolute magnitude is available, but those of intrinsically fainter stars are either in doubt or completely unknown. A provisional form for the luminosity function which affords a simple method of calculation gives general indications as to the behavior of space density, and most of the results thus far obtained have been derived in this way. But any results based on surface density are subject to large uncertainty and represent only the simplest interpretation of the data. The difficulty originates chiefly in the enormous differences in intrinsic stellar brightness, which ranges from about ten thousand times the luminosity of the sun to values only one ten-thousandth that of the sun.

Consider, for example, the space and surface densities in the direction of a star cluster of moderate density. The space density,

in general, decreases with increasing distance, but near the cluster shows a sharp and narrow maximum, whereas the surface density, because of the great dispersion in luminosity, reveals only an inconspicuous swelling extending over many magnitudes. In practice, we have only the broad, flat maximum in log N_m as a guide to the location of the irregularity in space density. In case several fluctuations occur at different distances along the line of sight, the surface density will include several broad, flat, overlapping maxima, and may show little variation with magnitude that can certainly be attributed to underlying irregularities in space density. A very flat maximum may be discernible in the counts, but, even so, its interpretation will be in doubt, for a single elementary maximum cannot be distinguished from the resultant of several maxima. In fact, a succession of fluctuations in space density may give values of log N_m which appear to indicate a perfectly regular and continuous decrease in density. For example, the stars in any direction might be shifted in distance so that all would lie on the surfaces of a series of concentric spheres, the radius of each sphere being twice that of the next inner sphere, without changing the apparent brightness of any star by more than three-fourths of a magnitude. The surface distribution corresponding to this discontinuous arrangement could not, however, in any practical problem, be distinguished from that actually observed, or better say, from that corresponding to a space distribution in which there were no fluctuations, but only a progressive change in space density. In most cases a continuous decrease is what is accepted as the result of calculations based on surface density; but it must be recognized that other interpretations are possible.

Highly concentrated groups like globular clusters of course show at once in the counts of stars, and the probability that two or more irregularities of this type should be exactly in line with the observer is small; but the alignment of larger aggregations, like the local cluster of helium stars or the star clouds of the Milky Way, must certainly be reckoned with. It may thus happen that large and more or less isolated aggregations of stars exist without our being able to detect them. Groups and clusters of stars that we actually see reveal themselves largely by contrast in surface density with adjacent regions of the sky. In the case of large aggregations, scat-

tered more or less at random, the effect of contrast is greatly reduced or wholly obliterated by the size of the star clouds and the overlapping of their boundaries. Such clouds may wholly lose their identity because the fluctuations in space density cannot be disentangled from the slender indications afforded by the star counts.

In studying the structure of the galactic system, however, the problem of space density must be faced eventually, in some form or other; but the preceding paragraphs indicate the importance of deferring its consideration to the last possible moment. Fortunately, much can be learned before the space density itself need be introduced, for the surface density shows important correlations with galactic latitude and longitude, as well as with apparent magnitude, and the discussion of these relations leads to important conclusions.

The approximately spheroidal symmetry in the values of log N_m with respect to the galactic plane has long been known, and has been expressed in the mean distribution tables of Kapteyn, van Rhijn, and others, and, finally, in those of *Contribution* No. 301. At the same time, it has also been known that the richest regions of the sky are in the general direction of Sagittarius. More recently Nort's discussion of the Harvard "Map of the Sky" showed that the periodic variation in surface density extends some distance on either side of the Milky Way, and, finally, the investigation in *Contribution* No. 346 confirmed the periodic fluctuation of surface density and showed it to be a conspicuous feature of stellar distribution which can be traced into high galactic latitudes and over a wide range of apparent magnitude. Each step in the accumulation of observations on the surface distribution of stars, beginning with the gauges of the elder Herschel, has led to conclusions bearing on the form and extent of the system, and we are now to consider from this standpoint the results collected in *Contribution* No. 346.

In this *Contribution* it was shown that the distribution between galactic latitudes 70° N. and 70° S. exhibits a systematic irregularity in longitude which may be represented by

$$\Delta = a + b \cos (\lambda - L') \, , \qquad (1)$$

where

$$\Delta = \log N' - \overline{\log N'} \, . \qquad (2)$$

N' is the observed number of stars per square degree at the point λ, β, and $\overline{\log N'}$, the average, for all longitudes, of the values of $\log N'$ in latitudes $+\beta$ and $-\beta$ as given in Table XVII, *Contribution* No. 301,[1] or in Table XIV, *Contribution* No. 346.[2] The parameters a, b, and L' for limiting magnitudes 9, 11, 13.5, 16, and 18, and for 10° intervals in latitude, are listed in Table XVI, *Contribution* No. 346.

The values of a are small; b decreases with increasing latitude, but is approximately the same for equal latitudes north and south; for a given m, the longitude of maximum density, L', is much the same for all latitudes. Deviations from the mean distribution having these general characteristics would follow were the sun displaced from the center of a stellar system having spheroidal symmetry. For this case, L' would represent the longitude of the center of the system as seen by the observer; the amplitude b would depend on the displacement of the sun parallel to the galactic plane, while a would be determined partly by the displacement perpendicular to this plane and partly by a possible systematic correction required by the adopted values of the mean distribution.

The values of b and L' for the two hemispheres show, however, certain systematic differences which suggest that the deviations used to calculate a, b, and L' may be affected by an error in the adopted position of the galactic pole, which was that of Gould. To test these suggestions numerically, we need formulae expressing the theoretical deviations from the mean distribution resulting from an eccentric position of the sun and from an error in the position of the galactic pole. These are developed in the following sections.

2. FORMULAE FOR THE ECCENTRIC POSITION OF THE SUN

Let the distribution of stars in all longitudes in a spheroidally symmetrical system be defined by

$$\log N_c = f(\sin^2\beta) \ , \tag{3}$$

where N_c is the number of stars per square degree brighter than m, in latitude β, as seen from the center of the system. Let $\log N$ be

[1] *Astrophysical Journal*, **62**, 320, 1925.

[2] *Ibid.*, **67**, 1928.

the density in the direction λ, β for the eccentrically located observer, and $\log N_0$ the mean of these densities averaged over all longitudes in latitudes $+\beta$ and $-\beta$.

We require an expression for

$$\Delta = \log N - \log N_0$$

as a function of the longitude and the constants expressing the characteristics of the stellar system and the position of the sun. This expression differs from (2) in that $\log N$ and $\log N_0$ refer to true latitudes in a perfectly symmetrical system, whereas $\log N'$ and $\overline{\log N'}$ represent the observed distribution, affected by local irregularities and other departures from symmetry, and by error in the position of the galactic pole.

The justification for the use of (3) lies in the fact that a symmetrical expression of this form represents, approximately at least, the observed mean distribution in the actual stellar system. It is therefore applicable to the representation of $\log N_0$ and, hence also to the representation of $\log N_c$, for later it will appear that the mean density differs from the central density only by a small quantity of the second order, and, further, that any disturbance arising from error in the pole is likewise of second order.

The variation of $\log N$, or of Δ, with longitude, depends not only on the eccentric position of the sun, but also on the space-density function $\Delta(\rho)$. As an indication of the method to be used for the further discussion of the observational data, it will be sufficient to determine the variation for the simple case of constant space density. For the supposedly symmetrical system of stars brighter than m, which gradually thins out with increasing distance, we accordingly substitute another system, sharply bounded and of constant density, but still defined by the symmetrical function (3).

Any radius of this system will be proportional to the cube root of the number of stars per unit angular area seen in the direction of the radius in question. With proper units, we may write

$$r_c = \sqrt[3]{N_c} \,, \tag{4}$$

where N_c is defined by (3).

Let Figure 1 represent the meridional section of the system through the sun at S. As fundamental plane for rectangular co-ordinates, we adopt the galactic plane GH, perpendicular to the diagram. The positive x-axis is directed toward H, in galactic longitude L, which is also the longitude of the center C as seen by the

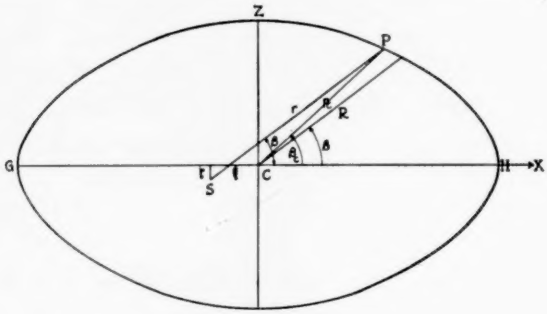

FIG. 1.

observer. The co-ordinates of S are denoted by $-\xi$, o, $-\zeta$; hence, those of C referred to S are ξ, o, ζ.

For any point P on the surface (P lies in the plane of Fig. 1, but the formulae are general), the rectangular co-ordinates x_c, y_c, z_c, referred to C, are connected with the co-ordinates X, Y, Z, referred to S by the relations,

$$x_c=x-\xi\ , \qquad y_c=y\ , \qquad z_c=z-\zeta\ . \tag{5}$$

Further, in terms of galactic co-ordinates λ and β,

$$\left.\begin{array}{ll} x_c=r_c\cos\beta_c\cos(\lambda_c-L) & x=r\cos\beta\cos(\lambda-L) \\ y_c=r_c\cos\beta_c\sin(\lambda_c-L) & y=r\cos\beta\sin(\lambda-L) \\ z_c=r_c\sin\beta_c & z=r\sin\beta \end{array}\right\} \tag{6}$$

Since the space density is assumed to be constant, we have for the observer at S, by analogy with (4),

$$r=\sqrt[3]{N}\ , \tag{7}$$

where N is the number of stars seen in the direction of P, in observed latitude β.

The relation of N to N_c for the same latitude may be found by determining the relation of r to R, R being the radius of the system for the latitude β, which is the same as the observed latitude corresponding to r. N_c is then easily evaluated in terms of N_0, which thus gives the required relation between N and N_0.

From (5) and (6)

$$r_c^2 = r^2 + \xi^2 + \zeta^2 - 2r^2 t \ , \tag{8}$$

where

$$t = \frac{\xi}{r} \cos \beta \cos (\lambda - L) + \frac{\zeta}{r} \sin \beta \ . \tag{9}$$

From (8), to terms of the second order, inclusive,

$$\frac{r_c}{r} = 1 - t - \frac{t^2}{2} + \alpha^2 \ , \qquad \alpha^2 = \frac{\xi^2 + \zeta^2}{2r^2} \ , \tag{10}$$

and, to the same accuracy,

$$\log \left(\frac{r}{r_c}\right) = \text{Mod } (t + t^2 - \alpha^2) \ . \tag{11}$$

Again, by Taylor's theorem,

$$\left.\begin{aligned} \frac{r_c}{R} &= 1 + \epsilon D_1 + \frac{\epsilon^2}{2} D_2 \ , & \epsilon &= \beta - \beta_c \\ D_1 &= \frac{1}{R} \frac{dR}{d\beta} & D_2 &= \frac{1}{R} \frac{d^2 R}{d\beta^2} \end{aligned}\right\} \tag{12}$$

D_1^2

from which

$$\log \left(\frac{r_c}{R}\right) = \text{Mod } (\epsilon D_1 + \epsilon^2 D_3); \quad D_3 = \frac{1}{2} (D_2 - D_1) \ . \tag{13}$$

Adding (11) and (13), and noting that

$$\left(\frac{r}{R}\right)^3 = \frac{N}{N_c} \ ,$$

we find

$$\log N - \log N_c = 3 \text{ Mod } (t + t^2 - \alpha^2 + \epsilon D_1 + \epsilon^2 D_3) \ . \tag{14}$$

The quantities D_1 and D_3, which depend only on R and its derivatives with respect to β, are independent of λ.

To find an expression for $\epsilon=\beta-\beta_c$, we use

$$r \sin \beta-\zeta=r_c \sin \beta_c \ . \tag{15}$$

Since

$$\sin \beta_c=\sin \beta+\epsilon \cos \beta-\frac{\epsilon^2}{2} \sin \beta+ \ldots \ ,$$

(15) may be written

$$\frac{r}{r_c}\left(\sin \beta-\frac{\zeta}{r}\right)=\sin \beta+\epsilon \cos \beta-\frac{\epsilon^2}{2} \sin \beta \ .$$

Substituting from (10),

$$\frac{r}{r_c}=1+t+\frac{3}{2} t^2-\alpha^2 \ , \tag{16}$$

and solving by approximations, we find

$$\epsilon=t_1+\left(\frac{t_1^2+3t^2}{2}-\alpha^2\right) \tan \beta-\frac{t\zeta}{r \cos \beta} \ , \tag{17}$$

where

$$t_1=\frac{\xi}{r} \sin \beta \cos (\lambda-L)-\frac{\zeta}{r} \cos \beta \ . \tag{18}$$

The right-hand members of (14) and (17) still contain the factor $1/r$, which is a function of λ. From (12) and (16),

$$\frac{1}{r}=\frac{1}{R}-\frac{t+\epsilon D_1}{R} \ . \qquad \text{(to 1st order)} \tag{19}$$

The substitution of (9), (17), (18), and (19) into (14) then gives

$$\log N-\log N_c=3 \text{ Mod } (t+t_1 D_1)+S \ , \tag{20}$$

where the second-order terms have the form

$$S=s_0+s_1 \cos (\lambda-L)+s_2 \cos 2(\lambda-L) \ . \tag{21}$$

$$S = 3\,\mathrm{Mod.}\left(\left[\frac{\xi}{R}\cos\beta\cos(\lambda-L)+\frac{\zeta}{R}\sin\beta\right]^2-\alpha^2+D_1\left\{\frac{3\left[\frac{\xi}{R}\cos\beta\cos(\lambda-L)+\frac{\zeta}{R}\sin\beta\right]^2+\left[\frac{\xi}{R}\sin\beta\cos(\lambda-L)-\frac{\zeta}{R}\cos\beta\right]^2}{2}-\alpha^2\right\}\tan\beta-\frac{D_1\zeta}{R\cos\beta}\left[\frac{\xi}{R}\cos\beta\cos(\lambda-L)+\frac{\zeta}{R}\sin\beta\right]+ \ldots \frac{\zeta}{R} \ldots\right.$$

Finally, with the aid of (9) and (18),

$$\log N = \log N_c + G + F \cos(\lambda - L) + s_0 + s_2 \cos 2(\lambda - L) , \qquad (22)$$

where

$$\left. \begin{aligned} G &= 3 \text{ Mod } (\sin \beta - D_1 \cos \beta) \frac{\zeta}{R} \\ F &= 3 \text{ Mod } (\cos \beta + D_1 \sin \beta) \frac{\xi}{R} + s_1 \end{aligned} \right\} \qquad (23)$$

The coefficient D_1 is proportional to the derivative of R with respect to β and changes sign with β. For the same latitude, north and south, G therefore has the same numerical value, but opposite signs. The average value of log N for latitudes $+\beta$ and $-\beta$ and all longitudes is therefore

$$\overline{\log N} = \log N_0 = \log N_c + \sigma_0 , \qquad (24)$$

where σ_0 includes the second-order terms. Hence, as stated above, the mean distribution differs from the surface density referred to the center only by second-order terms in ξ/R and ζ/R.

The substitution of (24) into (22) gives the required expression for Δ.

3. ERROR IN POSITION OF GALACTIC POLE

Let L_0 and p be the longitude and angular distance of the true pole P' relative to the adopted pole P; and let β' and β be the true and the adopted latitude, respectively, of a field F. From the triangle $PP'F$,

$$\sin p \cos(\lambda - L_0) = \sin \beta' \cos \beta - \cos \beta' \sin \beta \cos E ,$$

E being the angle at F. The side p is small, and, except for fields near the pole which are not here considered, the error in the adopted latitude may be written

$$\delta\beta = \beta' - \beta = p \cos(\lambda - L_0) . \qquad (25)$$

The observed density log N' is assigned to the incorrect latitude β. Compensation may be made by retaining β and correcting the density, whence the true density corresponding to β becomes

$$\log N = \log N' + D d\beta , \qquad (26)$$

in which D is the derivative of log N' for the mean latitude

$$\frac{1}{2}(\beta'+\beta)=\beta+\frac{\delta\beta}{2} .$$

For data collected according to the adopted constant latitude β, the corresponding true latitudes, and hence also D, vary with λ. We may write

$$D=D_0+\frac{\delta\beta}{2}\frac{dD_0}{d\beta} , \tag{27}$$

where D_0 and its derivatives refer to the adopted latitude β. Substituting (25) and (27) into (26), we have for the true density at latitude β

$$\log N=\log N'+k \cos (\lambda-L_0)+Q+Q \cos 2(\lambda-L_0) , \tag{28}$$

where

$$k=pD_0 , \qquad Q=\frac{p^2}{4}\frac{dD_0}{d\beta} . \tag{29}$$

From (28), averaged throughout all longitudes,

$$\overline{\log N}=\log N_0=\overline{\log N'}+Q . \tag{30}$$

The mean of the densities in the true latitudes β', all of which have been assigned to the adopted constant latitude β, therefore differs from the true mean density in this latitude only by the second-order quantity Q.

4. COMBINED EFFECT OF ECCENTRIC POSITION OF SUN AND ERROR IN GALACTIC POLE

The formulae of sections 2 and 3 are now to be combined in accordance with (2) to find an expression for Δ. Since the result is to be compared with the observational data, it should be noted that the values of the mean distribution in Table XVII, *Contribution* No. 301, which were used in forming the deviation Δ, are to be identified with $\overline{\log N'}$ in (30), and, further, that these tabular values may require a systematic correction s', because the original counts of stars were

not in all cases uniformly spaced in longitude. Introducing this correction and combining (30) with (24), we find

$$\overline{\log N'} = \log N_c + \sigma_0 - Q - s' \; . \tag{31}$$

The elimination of log N from (28) and (22) and substitution into (2) then gives

$$\Delta = s + G + F \cos (\lambda - L) - k \cos (\lambda - L_0) + s_2 \cos 2(\lambda - L) - Q \cos 2(\lambda - L_0) \; , \tag{32}$$

where the systematic correction s' has been combined with a residual second-order term $s_0 - \sigma_0$ and denoted by s.

The constant term Q has disappeared, and the second harmonics need not be retained, for it is easily seen that their neglect does not affect the numerical solution for the first harmonics, in which we are primarily interested. Moreover, the neglected second harmonics are of the second order.

We now distinguish between the two hemispheres, noting that sin β changes sign, and, by (23) and (29), that G and k also change sign. Except for the second-order term s_1, the value of F is the same in both hemispheres. Neglecting this difference, which is very small, we then have, for sin β always positive, ?

$$\left.\begin{aligned} \Delta_1 &= s + G + F \cos (\lambda - L) - k \cos (\lambda - L_0) \quad \text{(North)} \\ \Delta_2 &= s - G + F \cos (\lambda - L) + k \cos (\lambda - L_0) \quad \text{(South)} \end{aligned}\right\} \tag{33}$$

The substitutions

$$2s = a_1 + a_2 \; , \qquad 2G = a_1 - a_2 \tag{34}$$

and

$$\left.\begin{aligned} 2F \sin L &= b_2 \sin L_2 + b_1 \sin L_1 \; , \quad 2k \sin L_0 = b_2 \sin L_2 - b_1 \sin L_1 \\ 2F \cos L &= b_2 \cos L_2 + b_1 \cos L_1 \; , \quad 2k \cos L_0 = b_2 \cos L_2 - b_1 \cos L_1 \end{aligned}\right\} \tag{35}$$

reduce (33) to

$$\left.\begin{aligned} \Delta_1 &= a_1 + b_1 \cos (\lambda - L_1) \quad \text{(North)} \\ \Delta_2 &= a_2 + b_2 \cos (\lambda - L_2) \quad \text{(South)} \end{aligned}\right\} \tag{36}$$

which are of the form of (1).

It should be noted that this result does not depend on the law of space density. The assumption of constant density affects only the manner in which the co-ordinates of the center of the system, ξ and ζ, enter into F and G. It is only when we attempt to determine ξ and ζ from (23), or from the corresponding formulae for other density laws, that the special assumption for density comes into operation.

The hypothesis of a symmetrical stellar system and an eccentrically located sun, together with an erroneous value for the position of the galactic pole, therefore leads to deviations from the mean distribution which, to first-order terms, are of the same form as those given by observation. The obvious inference is that, tentatively at least, the observed distribution of the stars may be interpreted in accordance with this hypothesis, and we therefore proceed to the calculation of the quantities which define the distribution.

5. PARAMETERS FOR DEVIATIONS FROM MEAN DISTRIBUTION

By addition and subtraction, equations (33) give

$$\left.\begin{aligned} F\cos(\lambda-L)+s&=\tfrac{1}{2}(\Delta_2+\Delta_1)\\ k\cos(\lambda-L_0)-G&=\tfrac{1}{2}(\Delta_2-\Delta_1)\end{aligned}\right\}\qquad(37)$$

The sums and differences of the Δ's for the same longitude and the same latitude, north and south, therefore define cosine curves whose amplitudes and maxima depend, in the one case, only on the eccentric position of the sun, and, in the other, only on the error in the position of the galactic pole. Equations (37) may be used for least-squares solutions for the six parameters s, G, F, L, k, and L_0; but since the values of a, b, and L' are already known, the use of (34) and (35) is simpler. The polar distance p is found with the aid of k from the first of (29), which involves D_0, the derivative of log N_0, with respect to β. Values of this quantity have been calculated from the mean distribution and are collected in Table I.

The formulae are to be solved separately for each limiting magni-

tude for each pair of equal latitudes, north and south. For the special case $\beta=0$, G and k are both zero, and

$$\Delta_0=s+F\cos(\lambda-L)\ , \tag{38}$$

whence

$$s=a\ ,\qquad F=b\ ,\qquad L=L'\ . \tag{39}$$

For a homogeneous symmetrical system the parameters L, L_0, and p must have the same values for all latitudes and for all limit-

TABLE I

VALUES OF D_0, CHANGE IN LOG N_m PER 1° OF GALACTIC LATITUDE

(Unit=0.001)

β	m				
	9	11	13.5	16	18
5°......	9.4	11.6	14.4	14.8	14.2
10.......	13.4	14.4	17.2	21.2	24.0
20.......	11.8	12.6	15.8	22.0	29.2
30.......	9.8	10.4	12.8	16.0	20.9
40.......	7.0	7.9	10.3	13.6	15.7
50.......	5.1	6.3	8.7	11.5	12.5
60.......	4.0	5.3	7.5	9.6	10.4
70.......	3.1	4.1	5.8	7.1	8.0

ing magnitudes. The consistency of the values derived from different zones and from different magnitude limits will therefore provide an important test of the hypothesis on which the reduction is based. Should this test prove satisfactory, weighted mean values may be adopted for the three parameters.

To obtain weights for results derived from different zones of latitude, we note that values of a, b, and L' were calculated by least squares from the equation of condition

$$x\sin\lambda+y\cos\lambda+a=\Delta\ , \tag{40}$$

where

$$x=b\sin L'\ ,\qquad y=b\cos L'\ . \tag{41}$$

The form of the normal equations is such that the errors in x and y are equal. Denoting these errors by $d\delta$, we find from (35) and (41),

$$\left.\begin{aligned} dF &= (\cos L + \sin L)\, d\delta \\ FdL &= (\cos L - \sin L)\, d\delta \\ dk &= (\cos L_0 + \sin L_0)\, d\delta \\ kdL_0 &= (\cos L_0 - \sin L_0)\, d\delta \end{aligned}\right\} \tag{42}$$

and, finally, from the first of (29),

$$dp \propto d\delta / D_0 \; . \tag{43}$$

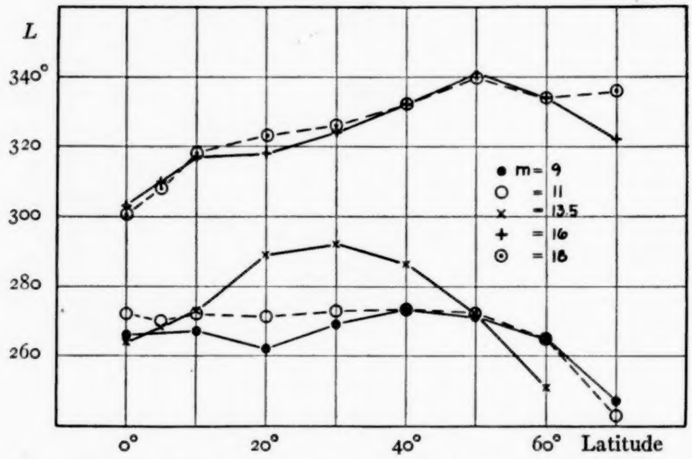

FIG. 2.—Longitude of center of system derived from surface densities in different zones of galactic latitude. Each curve corresponds to a different limiting magnitude.

Were the deviations of Δ from the cosine curves independent of the latitude, $d\delta$ would be constant; the several values of F and k would then be of equal weight, while the weights of L and L_0 would vary as F^2 and k^2, respectively. Figures 9–13, *Contribution* No. 346, show, however, that the irregularities in Δ are much larger near the Milky Way than in high latitudes. As a compromise, we weight L and L_0 according to F and k, respectively; and, since the large values of D_0 (Table I) in low latitudes will approximately compensate the correspondingly large values of $d\delta$, we assume dp to be constant and assign equal weights to the values of p.

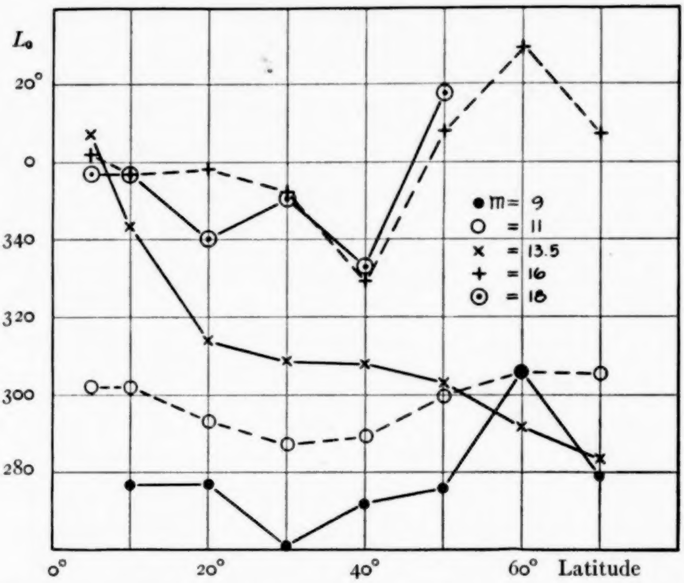

FIG. 3.—Longitude of north galactic pole derived from subsystems of stars brighter than a given limiting magnitude. The longitudes are referred to Gould's pole for the Milky Way clouds.

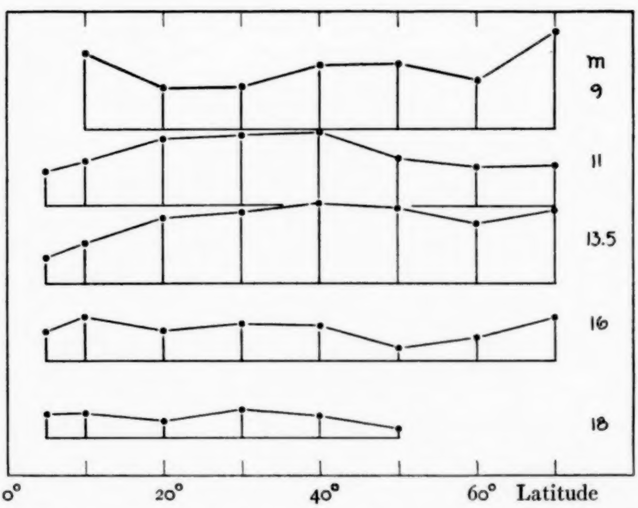

FIG. 4.—Distance between Gould's pole and galactic poles of subsystems defined by different limiting magnitudes. The interval between the axes corresponds to 10°.

TABLE II

Parameters and Star Ratios

β	F	L	L_*	k	p	δL	δL_*	δp	$\log Q_\xi$	$\log Q_\zeta$
					$m=9$					
0......	0.184	266°	.....	....	.....	+ 1°	.....	....	0.368	....
5......	....	....	.....	....	.....	.....	.....	....	....	....
10......	.199	267	277°	0.133	9°9	0	− 2°	− 1°8	.421	−0.097
20......	.190	262	277	.065	5.5	+ 5	− 2	+2.6	.419	.216
30......	.166	269	261	.055	5.6	− 2	+14	+2.4	.397	.205
40......	.156	273	272	.058	8.3	− 6	+ 3	−0.2	.397	.197
50......	.126	271	276	.040	8.5	− 4	− 1	−0.4	.373	.146
60......	.106	265	306	.025	6.4	+ 2	−31	+1.7	.413	.152
70......	0.038	247	279	0.039	12.6	+20	− 4	−4.5	0.234	−0.153
					$m=11$					
0......	0.222	272	.....	....	.....	− 2	.....	....	0.444	....
5......	.212	270	302	0.056	4.4	0	− 6	+2.4	.432	−0.082
10......	.214	272	302	.083	5.8	− 2	− 6	+1.0	.450	.100
20......	.200	271	293	.112	8.9	− 1	+ 3	−2.1	.438	.135
30......	.188	273	287	.097	9.3	− 3	+ 9	−2.5	.447	.145
40......	.153	273	289	.076	9.8	− 3	+ 7	−3.0	.397	.148
50......	.109	272	300	.039	6.2	− 2	− 4	+0.6	.338	.112
60......	.072	265	306	.026	5.0	+ 5	−10	+1.8	.337	.105
70......	0.033	243	305	0.021	5.1	+27	− 9	+1.7	0.236	−0.123
					$m=13.5$					
0......	0.197	264	.....	....	.....	+11	.....	....	0.394	....
5......	.232	268	7	0.046	3.2	+ 7	−48	+4.8	.475	−0.227
10......	.218	273	343	.090	5.2	+ 2	−24	+2.8	.462	.228
20......	.169	289	314	.134	8.5	−14	+ 5	−0.5	.380	.139
30......	.166	292	309	.118	9.2	−17	+10	−1.2	.403	.107
40......	.122	286	308	.106	10.3	−11	+11	−2.3	.333	.102
50......	.093	272	303	.086	9.9	+ 3	+16	−1.9	.321	.107
60......	.027	251	291	.059	7.9	+24	+28	+0.1	.141	.123
70......	(0.025)	(112)	283	0.055	9.5	.....	+36	−1.5	0.263	−0.122
					$m=16$					
0......	0.347	303	.....	....	.....	+16	.....	....	0.694	....
5......	.343	310	2	0.056	3.8	+ 9	− 5	+0.3	.706	−0.034
10......	.308	317	357	.118	5.6	+ 2	0	−1.5	.674	− .064
20......	.246	318	358	.086	3.9	+ 1	− 1	+0.2	.599	− .018
30......	.190	324	352	.079	4.9	− 5	+ 5	−0.8	.473	+ .008
40......	.130	332	329	.062	4.6	−13	+28	−0.5	.388	− .012
50......	.109	341	8	.019	1.6	−22	−11	+2.5	.444	− .001
60......	.052	334	32	.030	3.1	−15	−35	+1.0	0.370	− .028
70......	0.052	322	7	0.034	5.6	− 3	−10	−1.5	....	−0.006

TABLE II—*Continued*

β	F	L	L_0	k	p	δL	δL_0	δp	$\log Q_\xi$	$\log Q_\zeta$
					$m=18$					
0......	0.340	301	.....	....	.....	+18	.....	....	0.680	....
5......	.328	308	357	0.042	3.0	+11	− 7	−0.3	.675	−0.011
10......	.327	318	357	.075	3.1	+ 1	− 7	−0.4	.736	− .040
20......	.292	323	340	.060	2.1	− 4	+10	+0.6	.824	.000
30......	.202	326	351	.081	3.9	− 7	− 1	−1.2	.576	− .010
40......	.184	332	333	.045	2.9	−13	+17	−0.2	.576	+ .001
50......	.154	340	18	.015	1.2	−21	−28	+1.5	0.630	+ .029
60......	.111	334	(104)	.038	(3.7)	−15	.....	....	....	− .003
70......	0.078	336	(177)	0.017	(2.1)	−17	.....	....	....	+0.002

The values for F, L, L_0, k, and p found from the data for a, b, and L' in Table XVI, *Contribution* No. 346, are in the second to the sixth columns of Table II above. Those for s and G are in Table XVII of *Contribution* No. 346, and need not be repeated here. The results for L, L_0, and p are also shown graphically in Figures 2, 3, and 4.

Although some of the quantities, notably L for $m=13.5$, 16, and 18, and L_0 for $m=13.5$, show conspicuous progressive changes with latitude, the corresponding systematic deviations in the Δ's are

TABLE III

MEAN PARAMETERS AND STAR RATIOS

m	L	L_0	p	δL	δL_0	δp	$\log Q_\xi$	$\log Q_\zeta$
9.....	267°	275°	8°1	±4°0	±6°8	±1°9	0.396	−0.171
11.....	270	296	6.8	2.6	6.4	1.9	.435	− .127
13.5...	275	319	8.0	9.9	19.3	1.9	.423	− .112
16.....	319	357	4.1	8.9	9.4	1.0	.668	− .008
18.....	319	350	2.7	±9.7	± 9.9	±0.7	0.697	+0.004

small and confined mostly to the northern galactic hemisphere. As far as results for a single limiting magnitude are concerned, the agreement for different latitudes is such as to justify the formation of mean values for L, L_0, and p. These, together with their average deviations for a single zone, are in the second to the seventh columns of Table III.

6. DEPARTURE FROM SPHEROIDAL SYMMETRY

Thus far we have considered only first harmonics. The consistency of the results derived from different latitudes shows clearly the necessity of a correction to the galactic pole and supports the

TABLE IV*a*

RESIDUALS IN LOG N_m

(O−C, Unit=0.01)

$m=9$

λ	+70°	+60°	+50°	+40°	+30°	+20°	+10°	0°	−10°	−20°	−30°	−40°	−50°	−60°	−70°
0°	4	7	0	4	4	3	12	24	20	14	4	1	3	1	1
10	4	5	5	4	4	4	0	14	15	8	2	4	6	7	2
20	3	5	5	9	8	6	2	6	5	2	0	5	6	8	5
30	2	5	4	9	5	3	3	2	8	5	1	4	4	5	5
40	1	3	2	6	2	2	6	6	12	4	1	5	3	2	3
50	0	2	0	3	1	7	11	10	15	5	0	4	4	2	1
60	0	1	3	2	5	10	13	14	14	8	6	6	2	2	6
70	1	5	7	2	2	8	20	19	12	10	8	8	2	2	5
80	1	10	2	3	2	6	16	20	12	11	9	6	1	2	6
90	1	15	3	0	2	2	9	14	8	12	13	3	0	0	6
100	1	5	8	2	1	0	5	9	2	10	13	2	2	3	5
110	1	6	3	6	1	1	0	4	4	9	12	3	4	5	4
120	0	2	4	10	2	3	1	3	10	7	8	6	5	6	1
130	1	1	3	9	3	1	6	11	17	3	7	8	7	7	2
140	2	4	1	6	4	2	11	18	21	14	18	11	8	3	4
150	3	6	1	4	6	6	13	15	13	17	17	10	7	2	7
160	2	4	1	0	7	8	4	8	11	7	5	3	2	6	6
170	3	3	2	1	3	11	7	6	7	4	1	1	3	7	4
180	3	4	5	5	0	7	3	11	6	1	4	6	4	6	1
190	3	6	7	10	3	0	9	13	4	2	2	2	6	3	2
200	2	4	2	3	0	4	13	12	1	6	0	2	1	1	1
210	2	3	0	2	4	7	8	1	8	3	2	2	0	2	1
220	3	2	1	4	7	3	3	11	8	5	5	2	1	5	1
230	2	2	1	6	4	5	18	27	16	8	6	3	2	3	0
240	2	3	1	7	1	14	12	5	3	8	8	5	1	2	0
250	0	2	1	2	13	11	7	6	10	4	6	5	2	1	0
260	1	1	0	1	10	11	5	9	9	1	2	2	6	1	1
270	2	0	0	3	3	11	3	11	9	1	0	0	1	2	1
280	4	2	1	3	3	8	0	14	9	5	2	2	4	4	2
290	7	3	2	1	4	3	3	4	11	8	4	2	3	3	1
300	9	4	3	4	5	2	2	4	3	9	4	1	3	1	3
310	10	7	4	2	1	7	3	12	9	8	7	4	3	1	4
320	8	5	3	1	0	4	6	10	0	6	9	6	3	1	0
330	6	5	2	1	2	11	8	3	13	8	3	5	3	2	2
340	3	4	3	1	4	10	21	18	8	5	6	5	2	2	4
350	3	2	1	2	2	8	25	26	30	18	7	3	1	1	3

hypothesis of an eccentric location for the sun. It also shows that, as far as these harmonics are concerned, the system is nearly symmetrical. Irregularities associated with higher harmonics are not, however, excluded, and these are still to be investigated. Here a

distinction must be made between (1) actual deviations from spheroidal symmetry and (2) higher harmonics which are a consequence of the eccentric position of the sun and the error in the galactic pole and occur even in case the system is spheroidal. Terms

TABLE IV*b*

$m=11$

λ	+70°	+60°	+50°	+40°	+30°	+20°	+10°	+5°	0°	−5°	−10°	−20°	−30°	−40°	−50°	−60°	−70°
0°...	6	7	2	3	5	7	14	23	32	33	28	18	3	3	7	5	1
10....	5	8	7	2	0	2	3	8	16	19	17	10	3	4	9	6	1
20....	3	6	12	7	4	4	3	0	4	8	6	5	2	0	4	4	3
30....	1	3	9	11	12	14	18	17	10	5	4	0	0	2	1	2	4
40....	0	0	5	10	17	18	21	21	20	20	19	4	4	3	2	0	2
50....	2	2	0	5	12	19	20	21	21	23	23	6	6	3	1	1	0
60....	3	4	2	0	2	6	14	17	19	21	19	5	5	4	1	2	3
70....	4	7	8	0	1	4	3	7	15	19	17	8	1	2	1	1	3
80....	5	7	9	0	1	4	2	1	4	11	11	16	2	0	0	2	3
90....	6	7	9	1	2	4	0	5	18	22	20	9	3	1	1	1	3
100....	3	6	7	1	2	4	1	10	11	14	15	7	0	1	2	1	2
110....	2	4	4	2	3	3	2	6	0	6	8	2	1	5	4	2	2
120....	1	1	3	3	2	4	14	16	12	6	3	3	4	8	5	3	1
130....	0	1	2	1	3	9	20	27	30	30	30	7	7	10	5	2	4
140....	2	2	3	5	0	10	19	27	28	28	35	10	13	9	4	0	7
150....	3	2	3	3	2	10	15	14	11	6	11	24	17	7	1	3	6
160....	4	3	4	2	2	2	3	3	2	3	2	16	8	3	2	5	6
170....	3	3	2	1	1	6	6	6	3	1	8	8	1	1	3	6	5
180....	3	2	2	3	2	7	4	0	5	6	5	2	1	1	4	4	3
190....	3	3	2	3	3	6	1	1	5	3	3	0	1	1	4	3	3
200....	3	3	2	1	1	0	7	10	12	5	1	2	2	1	4	4	2
210....	3	2	1	1	6	11	6	3	2	4	3	3	2	4	5	2	1
220....	2	2	0	6	12	8	2	7	8	7	6	5	7	4	3	1	0
230....	2	2	0	8	9	2	13	19	16	16	15	8	8	4	0	1	0
240....	1	1	1	3	4	7	11	13	13	14	14	17	10	3	2	2	1
250....	0	1	1	2	0	5	9	9	3	4	8	10	8	9	1	3	0
260....	1	0	2	2	1	2	7	7	3	4	7	6	6	6	2	4	1
270....	2	0	4	2	2	2	10	12	2	5	8	3	3	3	3	4	1
280....	3	0	4	1	6	6	8	13	12	1	8	1	3	1	4	4	1
290....	4	0	3	0	8	7	7	9	7	11	13	2	1	0	6	4	3
300....	6	0	1	2	2	4	4	2	5	5	4	10	3	1	5	4	3
310....	5	2	0	3	4	4	3	4	13	12	6	8	5	4	4	3	3
320....	5	4	0	2	7	10	1	10	10	4	5	5	1	2	2	2	1
330....	3	4	1	2	10	24	11	2	19	7	7	2	1	2	3	0	1
340....	5	3	3	4	10	19	23	17	13	12	7	2	1	2	2	1	1
350....	5	2	1	5	7	14	30	30	31	30	29	22	9	3	4	0	4

of the second type appear in the preceding developments, but have been omitted from the final formulae since they are of the second order.

This being the case, the question of spheroidal symmetry is easily tested by subtracting the calculated first harmonics from the

observed deviations given in Tables XV*a–e*, *Contribution* No. 346. The formulae required are (33), which are to be used with values of F from Table II and with the mean values of L, L_0, and p given in Table III. The terms $s+G$ and $s-G$ are respectively equal to a_1 and a_2, which are collected in Table XVI, *Contribution* No. 346.

TABLE IV*c*

$m=13.5$

λ	+70°	+60°	+50°	+40°	+30°	+20°	+10°	+5°	0°	−5°	−10°	−20°	−30°	−40°	−50°	−60°	−70°
0°...	4	4	1	0	8	5	5	22	44	52	44	17	1	8	11	5	0
10....	1	3	1	6	16	15	9	1	21	28	31	15	2	3	2	0	0
20....	1	2	7	10	25	17	11	5	9	22	7	1	4	3	2	2	0
30....	4	7	11	11	27	33	33	29	4	4	2	3	9	4	0	3	1
40....	6	17	17	12	14	34	32	30	22	20	14	1	3	0	2	3	1
50....	13	16	11	8	8	14	26	26	20	21	20	3	2	0	6	5	1
60....	14	5	2	1	6	4	15	19	15	21	20	9	3	3	6	8	1
70....	9	0	8	8	10	3	8	5	12	20	20	13	4	1	5	10	1
80....	7	3	7	12	5	2	11	26	11	21	20	16	6	1	9	9	0
90....	7	5	2	1	16	7	8	37	13	23	23	18	8	0	8	5	1
100....	7	6	10	0	1	3	22	33	12	24	24	19	9	0	9	1	2
110....	8	2	9	11	0	11	44	10	0	13	21	17	5	3	8	2	3
120....	11	1	0	15	12	7	15	12	7	7	8	5	4	10	2	4	4
130....	6	4	6	3	3	5	19	19	15	4	6	13	13	13	2	5	4
140....	0	4	19	15	10	4	16	18	24	18	25	27	20	7	6	4	4
150....	1	0	9	8	0	7	10	13	12	6	15	31	21	4	6	4	5
160....	2	0	3	1	9	9	2	4	6	3	6	16	13	3	3	3	4
170....	3	1	1	4	8	5	5	7	7	4	3	11	7	3	1	4	3
180....	4	2	1	5	5	0	10	9	8	10	4	10	6	0	4	8	5
190....	5	3	3	5	5	0	9	7	5	4	4	7	0	3	6	6	6
200....	6	3	4	4	5	2	8	8	7	7	6	0	1	2	3	4	4
210....	6	4	3	4	6	3	5	6	5	3	1	4	0	1	1	2	2
220....	5	3	4	5	5	2	5	4	1	3	1	5	4	3	0	2	1
230....	5	2	3	4	5	3	4	6	4	0	1	1	0	3	0	1	0
240....	4	1	3	4	4	0	7	8	5	0	1	3	2	2	2	0	1
250....	4	0	2	2	3	0	12	13	9	1	1	6	8	3	2	1	2
260....	3	1	1	0	3	0	9	13	10	1	2	5	6	1	1	1	3
270....	4	0	0	0	2	1	11	14	10	0	0	3	4	1	0	1	4
280....	4	1	0	3	4	5	12	13	7	4	1	6	1	2	1	1	4
290....	4	1	1	0	8	11	16	15	10	9	12	16	1	0	0	1	5
300....	5	0	1	1	3	9	14	13	4	5	5	11	2	1	0	0	5
310....	4	1	3	4	7	4	0	2	7	11	8	9	1	3	3	0	5
320....	5	2	4	7	12	12	4	0	1	4	0	8	2	5	2	5	5
330....	6	3	8	8	14	26	20	3	5	13	6	1	1	2	6	5	4
340....	6	3	7	9	15	30	39	12	12	10	19	6	1	1	0	1	4
350....	5	3	6	6	4	16	32	39	33	32	22	14	2	7	5	0	3

I am much indebted to Miss Joyner for this rather laborious computation, the results of which are given in Tables IV*a–e*. To indicate more clearly the distribution of irregularities, positive residuals, representing an excess of observed density, are printed in bold-face type, negative residuals in ordinary type.

These residuals are obviously systematic and in certain parts of the sky indicate an excess in density of 60 per cent or more over that of a spheroidal distribution, and defects of even larger amount. The more striking departures are readily identified with well-known

TABLE IV*d*

$m = 16$

λ	+70°	+60°	+50°	+40°	+30°	+20°	+10°	+5°	0°	−5°	−10°	−20°	−30°	−40°	−50°	−60°	−70°
0°....	**2**	7	**4**	**10**	7	23	43	78	115	71	49	21	8	4	4	**2**	**1**
10....	**3**	**3**	**5**	**10**	3	22	37	59	85	66	39	21	8	6	5	4	3
20....	**4**	**5**	**5**	**11**	**5**	5	25	32	33	31	30	17	6	3	5	5	4
30....	**4**	**6**	**5**	**7**	**9**	**6**	**11**	**6**	8	16	20	13	6	2	3	5	5
40....	**5**	**8**	**6**	**5**	**6**	**9**	**7**	**5**	**2**	**1**	6	7	4	1	2	6	5
50....	**2**	**7**	**6**	**6**	0	**5**	**4**	1	7	0	1	0	2	0	2	5	4
60....	1	**6**	**6**	**7**	**5**	3	1	2	6	2	1	**4**	0	4	4	5	3
70....	6	**2**	**6**	**8**	1	6	5	1	3	3	3	1	**1**	1	1	4	**4**
80....	6	**2**	**3**	**1**	5	9	4	3	2	1	1	**5**	**5**	**7**	**11**	**2**	**5**
90....	5	**1**	**2**	1	14	8	**4**	**4**	**4**	**4**	**4**	**12**	**14**	**11**	**8**	**7**	**4**
100....	0	**1**	0	**1**	**2**	**1**	6	13	11	**9**	**8**	**12**	**13**	**3**	**3**	**10**	**5**
110....	**3**	1	3	**1**	**2**	8	15	29	23	10	**10**	**11**	**10**	3	0	**11**	**5**
120....	**7**	0	**1**	**2**	**3**	11	19	15	16	3	8	19	4	7	**2**	**11**	**3**
130....	**6**	1	**2**	**3**	**3**	5	4	1	**2**	**3**	3	67	11	5	**3**	**9**	**3**
140....	**4**	1	**3**	**2**	**3**	**7**	**6**	**7**	**7**	**6**	2	47	13	5	**6**	**5**	**1**
150....	**1**	4	**1**	**1**	**1**	**5**	**12**	**8**	**6**	**5**	1	26	16	3	**7**	**1**	1
160....	4	7	4	4	**2**	**7**	**12**	**3**	3	1	4	1	8	0	**9**	0	2
170....	7	12	9	5	0	**8**	**10**	**3**	9	18	19	**3**	**6**	**6**	**10**	1	2
180....	7	9	9	1	**4**	**13**	**18**	**23**	**20**	**3**	10	**3**	**5**	**4**	**9**	2	1
190....	1	8	9	**5**	**10**	**12**	**15**	**15**	**14**	**12**	**4**	**1**	**3**	**3**	**4**	**2**	1
200....	**3**	5	10	**2**	**10**	**9**	**7**	**8**	**10**	**10**	**3**	**1**	**4**	**2**	**1**	**1**	2
210....	**3**	**1**	2	0	**3**	**1**	**6**	**10**	**10**	**8**	**3**	**9**	1	2	**4**	2	3
220....	**2**	**3**	**2**	4	3	8	6	**1**	**4**	**6**	**3**	**3**	**3**	4	7	**4**	4
230....	**2**	**4**	0	6	10	14	12	9	5	6	6	**9**	**16**	0	11	5	6
240....	**1**	**4**	3	6	8	11	**4**	**1**	**4**	1	14	18	12	**1**	14	7	6
250....	**1**	**4**	5	6	6	8	3	**8**	**25**	**35**	**23**	**2**	8	10	14	9	2
260....	**1**	**4**	6	6	6	6	**13**	**19**	**44**	**19**	3	21	10	5	**14**	7	0
270....	**2**	**4**	6	4	7	5	**5**	**15**	**18**	**3**	16	26	14	8	6	5	0
280....	**4**	**4**	6	3	6	6	**2**	2	0	**3**	**8**	**2**	17	5	7	5	0
290....	**5**	**6**	6	2	1	**6**	2	12	11	1	**10**	**3**	9	3	2	2	0
300....	**3**	**8**	3	4	**4**	**10**	8	19	16	6	**11**	**9**	**8**	**9**	**2**	**2**	**4**
310....	**2**	**2**	**2**	12	3	**16**	**9**	2	13	16	10	**18**	**12**	**14**	**6**	**6**	**6**
320....	5	**1**	**5**	10	23	18	16	2	0	15	**3**	**22**	**15**	**13**	**8**	**8**	**6**
330....	10	2	**7**	10	30	45	35	26	18	8	**5**	**21**	**14**	**1**	**9**	**7**	**5**
340....	10	8	**6**	5	18	40	55	37	26	22	9	**15**	**24**	**4**	**6**	**5**	**3**
350....	6	15	**3**	**6**	7	22	58	69	58	44	26	4	**2**	**3**	**1**	**2**	**1**

regions of condensation or obscuration; and less conspicuous features persisting throughout a considerable range in magnitude are doubtless real and structurally important. Quantitatively, however, all the residuals are still uncertain. In this connection it may be noted that the region of low density extending from the lower left-hand

corner upward to the right in Tables IV*a* and IV*b* is just south of and nearly parallel to the equator, which suggests that the zone correction for the brighter stars in declinations 0° to 10° S. may be in error. On the other hand, the generally good agreement of the

TABLE IV*e*

$m = 18.0$

λ	+70°	+60°	+50°	+40°	+30°	+20°	+10°	+5°	0°	−5°	−10°	−20°	−30°	−40°	−50°	−60°	−70°
0°....	5	9	11	13	4	31	57	101	154	88	42	9	4	1	1	3	1
10....	5	7	3	2	8	11	24	45	61	46	28	11	2	2	3	4	4
20....	7	8	1	0	3	7	3	18	33	25	16	5	5	11	9	8	5
30....	8	8	7	3	4	12	11	10	2	19	17	3	7	2	4	0	5
40....	8	8	9	5	1	4	16	8	10	13	2	3	7	6	10	6	4
50....	7	7	8	8	1	3	10	9	6	14	6	3	2	8	11	8	4
60....	6	4	7	7	3	3	4	4	5	5	13	1	2	1	11	5	3
70....	2	2	8	8	3	1	6	2	3	8	10	2	2	2	7	3	2
80....	2	3	8	6	1	3	6	9	12	9	4	6	0	6	5	1	1
90....	3	6	7	5	2	3	8	4	6	4	1	3	4	4	2	2	0
100....	1	3	4	6	2	1	10	14	10	1	5	9	8	1	1	3	0
110....	1	1	0	5	4	0	14	33	20	2	9	14	6	2	3	2	0
120....	0	2	3	4	4	5	16	21	14	6	2	7	3	2	3	2	5
130....	1	3	4	1	1	3	11	12	10	15	22	36	12	2	0	6	11
140....	4	5	5	0	2	2	2	2	7	6	2	3	5	1	4	11	13
150....	7	7	4	1	1	6	9	10	9	4	3	1	5	5	6	15	16
160....	9	10	4	1	2	7	1	5	5	4	1	5	5	6	5	14	13
170....	11	14	4	1	2	8	1	1	7	15	13	1	5	3	8	9	8
180....	9	10	5	2	0	9	19	32	28	10	5	4	8	9	7	4	3
190....	8	8	5	1	8	16	26	30	27	19	12	10	3	7	5	1	2
200....	5	5	6	3	8	8	10	15	18	12	7	5	2	4	2	1	9
210....	3	3	8	6	3	2	5	9	11	6	2	0	6	1	1	2	9
220....	1	2	1	5	5	6	4	0	3	3	1	0	3	6	5	4	6
230....	0	2	1	3	5	12	11	7	5	5	5	8	14	2	9	7	2
240....	0	2	2	3	3	10	3	1	2	3	15	28	8	1	12	11	3
250....	0	1	3	3	4	8	4	6	23	34	22	1	8	14	14	13	3
260....	1	2	3	3	6	7	10	30	43	16	3	26	10	8	16	12	4
270....	0	3	3	5	6	7	4	14	17	1	18	32	34	11	9	11	4
280....	0	2	4	6	4	7	6	2	0	3	7	4	20	10	12	10	6
290....	0	2	4	5	0	4	4	11	14	0	9	5	9	7	4	9	2
300....	1	2	3	4	5	7	11	23	19	8	4	7	7	6	1	3	2
310....	1	3	3	4	3	12	7	3	13	15	10	10	13	10	3	1	3
320....	1	3	1	4	7	14	10	5	2	0	3	18	15	8	6	4	3
330....	0	4	1	7	14	27	21	21	17	6	4	20	15	7	3	2	1
340....	1	5	2	1	0	15	28	29	32	20	10	23	23	10	4	0	0
350....	3	7	5	10	10	14	28	49	69	52	35	18	17	5	1	1	1

lower right-hand portions of Tables IV*d* and IV*e*, corresponding to declinations −30° to −70°, illustrates the small uncertainty involved in the method of reduction. The deviations in question are the result of two independent discussions of data from the *Harvard-Groningen Durchmusterung* for the same limiting magnitude, 16.86.

The general magnitude of the departure from spheroidal symmetry is best indicated by the mean deviations given in Table V, which are average values for each magnitude, collected according to latitude. The excessive negative deviations omitted in determining the values a, b, and L',[1] all of which are in low latitudes and clearly a consequence of obscuration, were also omitted in preparing Table V. In latitudes 40°–70° the average values for all magnitude limits correspond to 8 or 10 per cent in the numbers of stars per unit area. Toward the galactic plane the mean departure increases to 25 or 30 per cent, part of which, however, is certainly to be attributed to

TABLE V

AVERAGE RESIDUALS IN LOG N_m

Gal. Lat.	m					
	9	11	13.5	16	18	Mean
0°	±0.111	±0.121	±0.110	±0.101	±0.101	±0.109
5	....	.114	.129	.078	.099	.105
10	.090	.105	.126	.075	.083	.096
20	.064	.074	.092	.088	.082	.080
30	.046	.044	.064	.067	.061	.057
40	.040	.032	.042	.047	.048	.042
50	.028	.032	.042	.052	.051	.041
60	.036	.028	.032	.047	.052	.039
70	±0.028	±0.027	±0.037	±0.034	±0.039	±0.033

obscuring material rather than to irregularities in stellar distribution. As a whole, the average departure is approximately one-half the average deviation of the observed counts from the mean distribution table. Allowance for the assumed eccentric location of the sun has therefore introduced an important simplification in the representation of the data. There remain, however, clearly marked irregularities, so systematic in character that the assumption of spheroidal symmetry, although useful as an intermediate step, can be regarded only as a first approximation. Because of the character of the data, the smoothing necessarily introduced by the reduction process, and certain unavoidable systematic errors, a more detailed analysis of the remaining irregularities does not appear to be profitable at present.

[1] *Mt. Wilson Contr.*, No. 346; *Astrophysical Journal*, **67**, 1928.

7. SOME STRUCTURAL DETAILS

Although the deviations from spheroidal symmetry are considerable, they do not seriously disturb the agreement of the first harmonics for a given limiting magnitude derived from the data for different latitudes. This, however, only emphasizes the fact that both the longitude of the center and the position of the galactic pole vary with the magnitude. The indicated uncertainty in the

TABLE VI

OTHER VALUES OF LONGITUDE OF CENTER

Authority	Source	Limiting Magnitude	L
Walkey*	Chart stars	4.0	244°
Walkey	H.A. 50	6.5	240
Walkey	DM Catalogues	9.6	265
Walkey	Oe5–B9	6.5	238.7
Walkey	Oe5–B5	6.5	234.8
Charlier†	B0–B5	6±	236
Gerasimovič‡	B0–B5	6.7	226
Nort§	Harvard Map	11	275
Strömberg‖	F, G, K, M	5.5±	257
Shapley¶	Globular clusters	....	325
Kapteyn**	Stream motion	....	257

* *Monthly Notices*, **74**, 649, 1914. Of Walkey's results only those of highest weight or of special significance for this discussion are quoted.

† *Meddelanden från Lund*, Series II, No. 14, 1916.

‡ *Vierteljahrsschrift der astronomischen Gesellschaft*, **61**, 227, 1926.

§ *Recherches astronomiques*, **7**, Utrecht, 1917. The longitude given is that of maximum stellar density, although Nort finally obtains 326° for the direction of the system of stars brighter than the eleventh magnitude.

‖ *Mt. Wilson Contr.*, No. 144; *Astrophysical Journal*, **47**, 7, 1918. The longitude is the direction of the center, derived on the assumption that the preferential motions of these stars are rotational in character.

¶ *Mt. Wilson Contr.*, No. 157; *Astrophysical Journal*, **49**, 311, 1919.

** *Ibid.*, No. 230; *Astrophysical Journal*, **55**, 302, 1922. The positions of the vertices of the two streams by themselves are insufficient to distinguish between the two values 77° and 257°. Kapteyn adopted the former, but clearly the latter is to be preferred.

mean values of L, L_0, and p given in Table III is only $\pm 2°$, $\pm 3°$, and $\pm 0.°6$, respectively. The actual uncertainty is doubtless larger, but no reasonable allowance for systematic error will account for the conspicuous changes in these parameters.

This dependence on limiting magnitude is, moreover, confirmed by other results. Those bearing on the longitude of the center are given in Table VI, and the general increase in L with magnitude there revealed has already been noted by Shapley.[1] As for the position of the fundamental plane, the classic investigation of Gould[2]

[1] *Mt. Wilson Contr.*, No. 157; *Astrophysical Journal*, **49**, 311, 1919.

[2] *Uranometria Argentina*, Cordoba, 1879.

led to the position of the pole of the Milky Way clouds adopted as the standard reference point for this discussion, namely,

$$\alpha = 190\overset{\circ}{.}3 \qquad \delta = +27\overset{\circ}{.}3\ (1875)\ . \tag{44}$$

Gould's investigation also brought to light the belt of bright stars which bears his name, a feature already noted by Sir John Herschel. The pole of the belt is in $\alpha = 171°$, $\delta = +30°$. Referred to the pole of the Milky Way, its co-ordinates are

$$L_0 = 167° \ , \qquad p = 17° \ . \tag{45}$$

Of special interest is Gould's conclusion that the stars brighter than $m = 4.1$ congregate toward the belt, rather than toward the plane of the Milky Way. This, and other facts,

. . . . indicate the existence of a small cluster, within which our system is eccentrically situated, but which is itself not far from the middle plane of the Galaxy. This cluster appears to be of a flattened shape, somewhat bifid, and to consist of somewhat more than 400 stars, of magnitudes from the first to the seventh. . . . .[1]

Newcomb,[2] using much the same data as Gould, found for the pole of the Milky Way (including both branches) $\alpha = 191\overset{\circ}{.}1$, $\delta = +26\overset{\circ}{.}8$, which differs only $0\overset{\circ}{.}8$ from Gould's position.[3] Newcomb also determined the pole for several different classes of objects, among them the stars brighter than 2.5 and 3.5 mag., respectively. Of special importance is his result for all the lucid stars,[4] $\alpha = 180\overset{\circ}{.}0$, $\delta = +21\overset{\circ}{.}5$. Referred to the standard pole, this becomes

$$L_0 = 210° \ , \qquad p = 11°\ (m \lesseqgtr 6)\ . \tag{46}$$

[1] *Ibid.*, p. 369.

[2] "On the Position of the Galactic and Other Principal Planes toward Which the Stars Tend To Crowd," *Publications of the Carnegie Institution of Washington*, No. 10, 1904.

[3] Two circumstances led to the adoption of Gould's position rather than a mean of the results by Gould and Newcomb: first, Newcomb does not state the equinox to which his result refers, although the evidence points toward 1875; and, second, Kapteyn's very useful tables for galactic latitude, *Groningen Publication*, No. 18, are based on Gould's pole.

[4] *Op. cit.*, p. 22.

Charlier's[1] discussion of 751 B stars gave, for the corresponding pole, $\alpha = 184^\circ\!.3$, $\delta = +28^\circ\!.7$ (1900). His list includes, however, many stars fainter than $m = 7.0$, which, as shown by Shapley and Miss Cannon,[2] tend to congregate toward the plane of the Milky Way.[3] On this account, Shapley's[4] result for the B0–B5 stars brighter than $m = 5.5$, namely,

$$L_0 = 160^\circ\ , \qquad p = 12^\circ\ , \tag{47}$$

is more significant for the present comparison. Finally, it may be noted that the nearer diffuse nebulae, according to Hubble,[5] congregate about a plane having the pole

$$L_0 = 160^\circ\ , \qquad p = 20^\circ\ .$$

These various results are shown graphically by Figure 5, in which the progression in L, L_0, and p with increasing magnitude appears with much definiteness. The data for L supplied by the present discussion confirm the values by Walkey and by Nort for $m = 9.6$ and 11, respectively, and extend the sequence to much fainter stars. The additional data for the position of the pole connect the divergent values given by the bright stars and the Milky Way clouds. The agreement with existing data for all three parameters is excellent. The only exception is a result by Newcomb,[6] derived from a fragmentary comparison of *Durchmusterung* stars, which he found to be distributed symmetrically with respect to the Milky Way. This discordance probably arises from the well-known irregularities in the limiting magnitude of the *Durchmusterung* catalogues.

The progression in the parameters is obviously related to the grouping of bright stars investigated by Gould, which now appears as an important structural feature. The existence of a local cluster of bright helium stars, a few hundred parsecs in diameter, has al-

[1] *Meddelanden från Lund*, Series II, No. 14, p. 40, 1916.

[2] *Harvard Circular*, No. 239, 1922.

[3] Charlier's recent discussion in *op. cit.*, No. 34, 1926, also groups bright and faint stars together.

[4] *Mt. Wilson Contr.*, No. 157; *Astrophysical Journal*, **49**, 311, 1919.

[5] *Mt. Wilson Contr.*, No. 241; *Astrophysical Journal*, **56**, 162, 1922.

[6] *Op. cit.*

ready been amply demonstrated by Charlier[1] and by Shapley.[2] In fact, its presence is revealed by ordinary star counts, which show that these objects thin out with increasing distance much faster than the stars at large.[3] Some of the A stars have also been identified with this cluster.[4] The present results indicate that a considerable fraction of the fainter stars, even of the twelfth and fourteenth

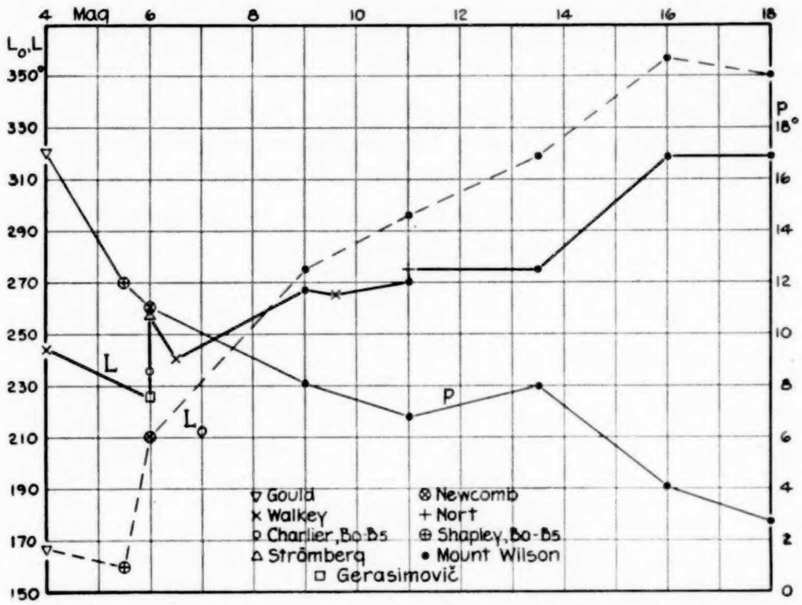

FIG. 5.—Dependence of longitude of center (L) and position of galactic pole (L_0, p) on limiting magnitude.

magnitudes, are involved and that the cluster is not only richer, but much larger, than hitherto supposed.

A question perhaps arises as to whether the group is really a separate dynamical unit. The isolated nucleus of helium stars suggests a separate aggregation whose influence on the observed distribution decreases with increasing magnitude, but the progressive change in the parameters which define the distribution may perhaps

[1] *Loc. cit.* [2] *Loc. cit.*

[3] *Publications of the Astronomical Society of the Pacific*, **30**, 114, 1918; *Adolfo Stahl Lectures* (San Francisco, 1919), p. 225.

[4] Shapley and Cannon, *Harvard Circular*, No. 229, 1922.

be analogous to the progression shown by Strömberg to be a striking feature of stellar motions. Provisionally, however, the concept of a local aggregation, but now of increased size and importance, is retained as the simpler interpretation.

The importance of the local cluster is illustrated by the fact that apparently it includes most of the bright stars. The fundamental plane defined by stars brighter than the fifth or sixth magnitude agrees closely with that of the nucleus of helium stars, which presumably is to be taken as the fundamental plane of the cluster as a whole. This agreement could not exist were any large fraction of the bright stars symmetrically distributed with respect to the plane of the Milky Way, which is inclined 12° to that of the cluster. If the agreement is really as close as it seems, it follows that the local cluster is of dominating influence in determining the space density near the sun.

Other details illustrated in Figure 5 confirm this inference and lead to an estimate of the relative densities of the cluster and of the larger system. For the limiting magnitude which corresponds approximately to median values of the parameters, the cluster and non-cluster stars should be equally numerous. The limit in question is 11–12 mag.; the mean parallax of the stars involved is of the order of $0\overset{''}{.}002$, and, since the mean distance is about $1.5/\pi$, 700 or 800 parsecs is roughly the distance at which the contributions of cluster and non-cluster stars should be equal. At this distance the total density in the galactic plane averages about one-half that near the sun, while the contribution of the larger system, averaged throughout all longitudes, must be approximately the same as its density at the sun. Hence, near the sun, the space density of the local cluster is about three times that of the larger system.

As for the size of the cluster, Figure 5 shows that its influence can be traced to the fifteenth magnitude at least. The mean distance of stars of this brightness is not well known, but it seems reasonable to suppose that stars belonging to the local cluster occur at distances of 3000 parsecs or more. This estimate of size assumes the spectral composition of the cluster to be comparable with that of the larger system, which seems probable in view of the fact that the local cluster contributes so largely to the space density near the sun.

8. RATIOS FOR SURFACE DENSITY AND TOTAL LIGHT

The agreement of the first harmonics derived from different zones of latitude is consistent with the hypothesis of an eccentrically located sun within an approximately symmetrical system. The dependence of the parameters on limiting magnitude and the resulting conclusion as to the importance of the local cluster show, however, that this simple hypothesis is of limited applicability.

This complicates the problem of locating the center of the system, which otherwise could easily be found with the aid of equations (23), since these formulae connect the co-ordinates ξ and ζ directly with F and G. Notwithstanding the dominating influence of the local cluster, (23) may be used to obtain values of the ratios of surface density and of total light for stars in the directions of center and anti-center. These ratios are important results, based directly on observational data and apparently little affected by the special assumptions used.

The procedure is to calculate for each latitude, with the aid of (23), the quantities

$$\left.\begin{aligned} Q_\xi &= \frac{\text{No. of stars in } \lambda=L,\ \beta=0^\circ}{\text{No. of stars in } \lambda=L+180^\circ,\ \beta=0^\circ} \\ Q_\zeta &= \frac{\text{No. of stars in } \beta=+90^\circ}{\text{No. of stars in } \beta=-90^\circ} \end{aligned}\right\} \tag{48}$$

which then lead to values of

$$\left.\begin{aligned} B_\xi &= \frac{\text{Total light of stars in } \lambda=L,\ \beta=0^\circ}{\text{Total light of stars in } \lambda=L+180^\circ,\ \beta=0^\circ} \\ B_\zeta &= \frac{\text{Total light of stars in } \beta=+90^\circ}{\text{Total light of stars in } \beta=-90^\circ} \end{aligned}\right\} \tag{49}$$

Under the conditions of the original hypothesis, the quantities B_ξ and B_ζ, combined with the space-density function which must be introduced at this point, would have given the co-ordinates of the center. This calculation will, in fact, be made, as though the conditions were those originally supposed. Although the center cannot be located by this method, the results strengthen the conclusions as to the importance of the local cluster.

The assumption of constant space density used in deriving equations (23) is retained, and any results vitiated by this assumption will be rejected later. It is thus possible to write

$$Q_\xi=\left(\frac{R_0+\xi}{R_0-\xi}\right)^3, \qquad Q_\zeta=\left(\frac{R_{90}+\zeta}{R_{90}-\zeta}\right)^3, \tag{50}$$

in which R_0 and R_{90} are the radii of this system for $\beta=0°$ and $90°$, respectively. The maximum value of ξ/R_0 is about 0.25; ζ/R_{90} is much smaller. Hence, with an error rarely exceeding 1 per cent,

$$\log Q_\xi=6 \text{ Mod } \frac{\xi}{R_0}, \qquad \log Q_\zeta=6 \text{ Mod } \frac{\zeta}{R_0}. \tag{51}$$

Since

$$R_0^3=N_0, \qquad R_\beta^3=N_\beta, \qquad R_{90}^3=N_{90},$$

in which the values of N are those corresponding to the mean distribution, and since, further,

$$D_1=\frac{1}{R}\frac{dR}{d\beta}=\frac{57.3}{3 \text{ Mod}}\frac{d \log N}{d\beta}=44\, D_0, \tag{52}$$

(51) and (23) then give

$$\left.\begin{aligned}\log Q_\xi&=\left(\frac{N_\beta}{N_0}\right)^{\frac{1}{3}}\frac{2F}{\cos\beta+44D_0\sin\beta}\\ \log Q_\zeta&=\left(\frac{N_\beta}{N_{90}}\right)^{\frac{1}{3}}\frac{2G}{\sin\beta-44D_0\cos\beta}\end{aligned}\right\} \tag{53}$$

The values of D_0 and F are in Tables I and II; G, in Table XVII of *Contribution* No. 346. The results derived from (53) are in the last two columns of Table II and are shown graphically in Figure 6.

For $\beta=0°$, $\log Q_\xi$ is directly twice the parameter F; for $\beta=90°$, $\log Q_\zeta=2G$. For all other latitudes the results depend on the assumption of constant space density, although for a considerable part of the sky they seem to be little affected by this incorrect assumption. In low latitudes Q_ξ is constant, or shows a slight increase; in high latitudes, it decreases most rapidly for the faint limiting magnitudes. The changes are partially obscured in Figure 6, where small

irregularities in F appear greatly magnified in some of the values for $m = 16$ and 18. A calculation with smoothed values of F shows, however, a steady and rapid decline in high latitudes.

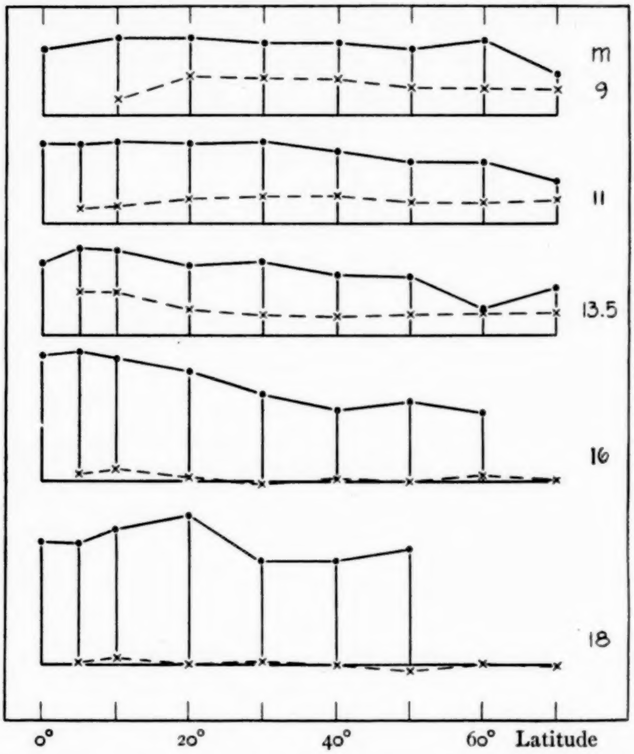

FIG. 6.—Logarithms of ratios of numbers of stars in directions of center and anticenter ($\log Q_\xi$, points) and of north and south galactic poles ($\log Q_\zeta$, crosses. For convenience these are plotted with reversed signs.) The progression in the ratios derived from different zones of latitude, especially noticeable in $\log Q_\xi$, is a consequence of having assumed that the space density is constant. Conversely, the progression is an indication that the space density decreases with increasing distance from the sun. Values of $\log Q_\xi$ from high latitudes and of $\log Q_\zeta$ from low latitudes are rejected.

Had the correct law of space density been used, the different values of Q_ξ would have been equal. As it is, they give a new proof of the long-known fact that the stars thin out with increasing distance from the center. Since the density actually decreases, the foregoing formula for $\log Q_\xi$ necessarily gives results which diminish

with increasing latitude. When the limiting magnitude is bright, the decrease should be small, since a large percentage of the stars will then be within the region of slowly changing density near the sun where the conditions approximate those upon which the formula is based. For faint limits, on the other hand, the decrease in high latitudes will be rapid; and should there be well-marked aggregations of stars close to the galactic plane, the decrease is likely to be preceded by a slight increase in low latitudes, similar to that shown for $m=18$.[1] The behavior of $\log Q_\xi$ is therefore only that to be expected.

For $\log Q_\zeta$ the conditions are somewhat different, principally because the displacement of the sun from the galactic plane is so small. In these circumstances, $\log Q_\zeta$ should be nearly constant, as in fact it is. Results from low latitudes, however, are of relatively low weight.

The combination of individual values of $\log Q_\xi$ and $\log Q_\zeta$ into means thus involves a compromise. To reduce the effect of accidental errors, the interval in latitude should be large, but not so large that the incorrect assumption for space density affects the results. An inspection of Figure 6 indicates that for $\log Q_\zeta$ the interval $30°$–$70°$ is probably safe, and that for $\log Q_\xi$ we may use

m	9	11	13.5	16	18
Interval	$0°-50°$	$0°-40°$	$0°-30°$	$0°-20°$	$0°-10°$

The corresponding means are given in the last two columns of Table III.

Both Q_ξ and Q_ζ depend on the limiting magnitude. For $m=9$ about 2.5 times as many stars are seen in the direction of the center as in the opposite direction. For the fainter limits, $m=16$ and 18, the corresponding ratio is about 5. The slight asymmetry in latitude, on the other hand, is most conspicuous for the bright stars, the ratio for the directions $+90°$ and $-90°$ being 2:3 for $m=9$ and increasing to unity for $m=16$.

The evaluation of B_ξ and B_ζ begins with the expression for the

[1] The calculation with smoothed values of F shows a similar increase for $m=16$.

light received from the stars of magnitude m, which, in terms of the light of a first magnitude star, is

$$l_m = \frac{dN_m}{dm} e^{\frac{0.4(1-m)}{\text{Mod}}} = \frac{N_m}{\text{Mod}} \frac{d \log N_m}{dm} e^{\frac{0.4(1-m)}{\text{Mod}}} . \tag{54}$$

For the directions of center and anti-center N_m is to be replaced by the maximum and minimum values of the surface density:

$$\left.\begin{aligned} \log N'_m &= \log N_m + \tfrac{1}{2} \log Q_\xi \\ \log N''_m &= \log N_m - \tfrac{1}{2} \log Q_\xi \end{aligned}\right\} \tag{55}$$

where $\log N_m$ still represents the mean distribution. Further, the derivative in (54) may be replaced by δ, the first difference in $\log N_m$. These substitutions give

$$\left.\begin{aligned} \log l'_m &= \log \frac{1}{\text{Mod}} + \log N_m + \tfrac{1}{2} \log Q_\xi + \log \delta + 0.4(1-m) \\ & \qquad\qquad\qquad\qquad\qquad\qquad (\textit{Center}) \\ \log l''_m &= \log \frac{1}{\text{Mod}} + \log N_m - \tfrac{1}{2} \log Q_\xi + \log \delta + 0.4(1-m) \\ & \qquad\qquad\qquad\qquad\qquad\qquad (\textit{Anti-center}) \end{aligned}\right\} \tag{56}$$

and for $\beta = +90°$ and $-90°$, two similar expressions involving Q_ζ.

To find the light received from all magnitudes, these formulae must be summed over all values of m. This requires an extrapolation of the values of Q_ξ and Q_ζ. For the bright stars the resulting uncertainty is not serious, because of the condition that for $m = 0$ the logarithms of both Q_ξ and Q_ζ are approximately zero. For a maximum luminosity of $M = -5$, all stars of zero apparent magnitude will be within a distance of 100 parsecs. The percentage change in space density within this distance, whatever the location of the solar system, must be small. The ratio of the number of stars of zero apparent magnitude in the direction of the center to the number in the opposite direction cannot therefore differ greatly from unity, and, in consequence, the logarithm of the ratio must be small. The extension of Q_ξ and Q_ζ to stars fainter than $m = 18$ likewise causes no difficulty, because the contribution of these stars is only a small fraction of the total light.

The integrated results from (56) and from similar formulae for the direction of the poles, combined in accordance with (49), give, finally,

$$B_\xi=3.04 , \qquad B_\zeta=1.29 . \tag{57}$$

The uncertainty resulting from the extrapolation is less than 0.1.

9. RELATION OF SPACE DENSITY TO TOTAL LIGHT

The light from the stars in a thin shell of radius ρ and volume V, referred to unit distance, is

$$dl=\frac{l_0}{\rho^2}\,\Delta(\rho)V ,$$

where $\Delta(\rho)$ is the space density, and l_0 the light from the stars in 1 cubic parsec of unit density at unit distance. The volume is

$$V=\omega\rho^2 d\rho ,$$

and l_0 is assumed to be independent of the distance. Integration then gives for the total light from the cone of angular area ω

$$l=\omega l_0 D , \tag{58}$$

where D is the integral of the space density from the point of observation to $\rho=\infty$.

For the observer on the earth the totals for the direction of center and anti-center are

$$l'=\omega l_0(D_0+d) , \qquad l''=\omega l_0(D_0-d) ,$$

where d and D_0 are the values of the integrated space density from the center of the system to the limits $\rho=\xi$ and $\rho=\infty$, respectively. Hence

$$B_\xi=\frac{l'}{l''}=\frac{D_0+d}{D_0-d} , \tag{59}$$

from which

$$\frac{d}{D_0}=\frac{B_\xi-1}{B_\xi+1} . \tag{60}$$

A similar expression involving B_ζ and D_{90} applies to the directions of the galactic poles.

Since from (57) $B_\xi = 3.04$, it appears that ξ has a value such that the density summed to that point from the center of the system is $0.50 D_0$. For the ζ co-ordinate the corresponding fraction is $0.13 D_{90}$.

10. THE CO-ORDINATES ξ AND ζ AND SOME GENERAL DISCUSSION

The final step in the determination of ξ and ζ by the method outlined requires the space-density function referred to the center of the system, for directions in and perpendicular to the galactic plane, and the integrals D_0 and D_{90}. This density function is unknown, but, as a first approximation, an attempt may be made to replace it by the space densities of Kapteyn and van Rhijn.[1] These densities are referred to the sun as a center and are mean values, averaged throughout all longitudes for $\beta = 0°$, and for northern and southern hemispheres in the case of $\beta = 90°$. The resulting values of the co-ordinates are[2]

$$\xi = 1000 \text{ parsecs} , \qquad \zeta = -30 \text{ parsecs} ,$$

where the negative sign indicates that the sun is north of the galactic plane. These results ignore the existence of the local cluster and are chiefly of value because they strengthen the conclusion that this important aggregation cannot be ignored.

The ζ co-ordinate.—Consider first the displacement from the galactic plane. The value found is in direct conflict with the star counts for limiting magnitudes 16 and 18. The results for G given in Table XVII, *Contribution* No. 346, which are one-half the differences in log N_m for the two galactic hemispheres, show that the total numbers of stars brighter than these limits are sensibly the same for the same latitude, north and south. The values of log Q_ζ in Table II above indicate the same thing, in a little different form, even more conclusively. The sun therefore appears to be almost exactly in the galactic plane defined by the faint stars. For sub-

[1] *Mt. Wilson Contr.*, No. 229, Table I*b*; *Astrophysical Journal*, **55**, 242, 1922.

[2] A rough calculation at an early stage of the discussion gave $B_\xi = 4$, and $\xi = 1200$ parsecs (*Mt. Wilson Report, Year Book of the Carnegie Institution of Washington*, No. 25, 1926; *Nature*, **119**, 461, 1927). In the meantime, the local cluster has been recognized as a feature of such magnitude as to alter radically the interpretation of the data.

systems having the limits $m=9$, 11, and 13.5, this is not the case. For these systems the sun is north of the respective fundamental planes, which do not coincide with the galactic plane for the fainter limits. This difference in behavior undoubtedly represents the influence of the local cluster, which lies to the south of the plane of the larger system. Judged by the values of G, or of $\log Q_\zeta$ in Table III, the cluster includes stars as faint as the fifteenth magnitude, which agrees well with the evidence of Figure 5.

Although relatively few in number, the bright stars contribute a considerable fraction of the total light received. Because of their asymmetrical distribution, the light-ratio deviates appreciably from unity, and the calculated values of ζ for the system as a whole is seriously in error. At the same time, because the bright stars are relatively few, they have only a negligible effect on the ratio of surface density Q_ζ, when the limiting magnitude is faint. A better indication of the position of the sun relative to the larger system is therefore given by the unit value of this ratio for $m=16$ and 18, to which corresponds $\zeta=0$.

The uncertainty of this result depends on the systematic errors affecting the star counts, which are unknown. Although the differences for the two hemispheres appear to be vanishingly small, the close agreement may be accidental. A constant systematic difference of 0.02 in $\log N_m$, which would correspond to a relative systematic error of from 0.05 to 0.08 in the magnitude scales, is probably more than a generous allowance. For such an error B_ζ would become 1 ± 0.05, with a corresponding uncertainty in ζ of about 8 parsecs.

The result $\zeta=0$ refers to a galactic plane defined by equality in the numbers of stars in northern and southern hemispheres and depends only on the observed equality in surface densities in latitudes 30°–70°. The stars in low latitudes, which are of little weight for the determination, are omitted. The space-density function has been used only to indicate the sensitiveness of the method and not at all to obtain the adopted result.

The close coincidence of the sun with the galactic plane of the larger system is in harmony with the very careful investigation by Gould, which, in his words,

. . . . justifies the distinct statement that, excepting from $15\frac{1}{2}^h$ to $19\frac{1}{2}^h$ of right ascension, where the branches [of the Milky Way] are separated, the medial line of the Galaxy is not distinguishable from a great circle.[1]

The actual systematic deviation[2] of Gould's adopted points is $6'$ to the north, but the inclusion of the bifurcated region would have given an equal deviation to the south. The points of Gould's "medial line" were determined[3] partly by the brightness of the Milky Way and do not coincide with the middle line between the borders, which was found to be a small circle, $51'$ north of the adopted great circle. This indicates something of the inherent uncertainty of the problem, which is emphasized by Newcomb's conclusion[4] that the central line of the galaxy is a degree to the south of his adopted great circle.

Gould's evidence does not prove that the displacement of the sun from the galactic plane is small; the conclusion is rather that his investigation does not contradict the small value found above. On the other hand, had Gould found an appreciable "dip" for the galactic circle, it would have been necessary to admit that the linear displacement might be considerable.

The other evidence bearing on the position of the sun relative to the galactic plane has been derived from special classes of objects. Thus Shapley[5] found the sun to be 60 parsecs north of the plane defined by the Cepheid variables, which are intimately associated with the clouds of the Milky Way. At the same time Shapley and Miss Cannon[6] find the sun to be only 15 parsecs north of the plane of the B0–B5 stars in the magnitude interval 7.26–8.25, which seem to be related to the primary galaxy rather than the local cluster.

With the aid of recent data the value from Cepheids has been revised to 34 ± 11 parsecs by Gerasimovič and Luyten,[7] who have also given closely agreeing values for *c* and *ac* stars, and for stars of spectral types O and B. Although some of these data may not be wholly free from the influence of the local cluster, there remains a

[1] *Op. cit.*, p. 370.
[2] *Ibid.*, p. 376.
[3] *Ibid.*, p. 382.
[4] *Op. cit.*, p. 18.
[5] *Mt. Wilson Contr.*, No. 157; *Astrophysical Journal*, **49**, 311, 1919.
[6] *Harvard Circular*, No. 229.
[7] *Proceedings of the National Academy of Sciences*, **13**, 387, 1927.

definite indication of systematic difference between the results from star counts and from special classes of objects. It should be noted, however, that, aside from different data, different definitions of the fundamental plane have been used—for example, equality in numbers as against equality in mean latitudes and distances. Since Gould's observations show that the distribution of brightness across the Milky Way is not symmetrical, it is perhaps to be expected that differences in method and definition will lead to slightly different results.

The ratios of surface density for the limits $m=9$, 11, and 13.5 indicate that the sun is north of the plane of the local cluster. The actual distance should be greater than the provisional 30 parsecs found for the system as a whole, which is a weighted mean for the local cluster and the larger system, but not much greater, because the faint stars of the larger system contribute only a small percentage of the total light, while the bright stars, which contribute largely, are mostly members of the cluster. Perhaps 40–50 parsecs may be taken as the result of these general indications, although the actual value is very uncertain.

These results place the local cluster an equal amount to the south of the plane of the larger system and lead to an expectation that the distance of the sun to the north of the plane of the bright B stars (belonging to the local cluster) should be greater than its distance from the plane of the faint B's (belonging to the larger system). Everyone[1] who has investigated the B stars finds that, as a group, they are south of the sun; but the expected difference depending on magnitude does not appear. In fact, Gerasimovič[2] finds the sun to be nearer the plane of the bright B's (16 parsecs) than that of the fainter B's (34 parsecs); and something qualitatively similar is suggested by the data of Shapley and Miss Cannon.[3] Although these data show that the dip is greater for bright stars than for faint, the reverse, if anything, seems to be true of the linear displacement.

Here again the indications from inclusive star counts apparently

[1] In addition to preceding citations, see Charlier, *op. cit.*, No. 14, p. 31, 1926, who finds the sun to be 15 parsecs north of the plane of 1238 B0–B5 stars brighter than $m=8.0$.

[2] *Vierteljahrsschrift der astronomischen Gesellschaft*, **61**, 227, 1926.

[3] *Harvard Circular*, No. 239.

differ from those afforded by a special class of objects. The proper apportionment of the discordances between such contributing causes as errors of observation or reduction and statistical peculiarities in the distribution of stars cannot now be made. Present results are therefore relevant only to the data and the method used in deriving them.

The ξ co-ordinate.—The displacement of 1000 parsecs has been calculated by disregarding the local cluster and assuming that the space density in the galactic plane relative to the center of the system may be replaced by the mean density relative to the sun obtained by averaging throughout all longitudes. Thus in Figure 7 the curve represents the mean densities of Kapteyn and van Rhijn, with the maximum shifted to coincide with the center of the system C. The location of the sun satisfies the condition $d = 0.50D_0$. But

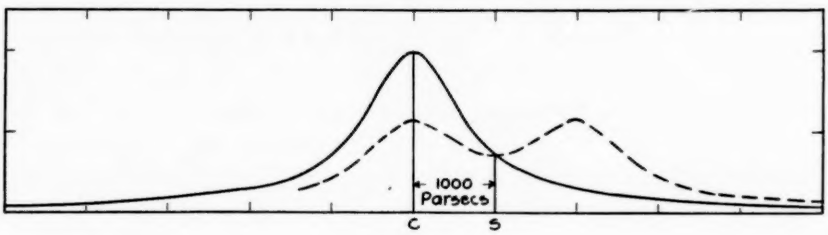

FIG. 7

at S, 1000 parsecs from C, the mean density is only 0.35 that at the center. This immediately invalidates the assumption that the actual distribution of space density can closely resemble that shown. For were the assumption justified, the results obtained by folding the figure about the vertical line through S and averaging overlapping ordinates would agree approximately, at least, with the mean densities of Kapteyn and van Rhijn. Actually this is not the case; the average of the ordinates gives a curve (dotted curve, Fig. 7) showing a minimum at S, a slight increase to a maximum, and finally a decline to small densities, but only at relatively great distances. Further, the hypothesis underlying the calculation of ξ would require a large increase in the space density in the direction of the center for a thousand parsecs or so. It is easily shown, however, with the aid of ordinary star counts and without any elaborate calculation, that no such increase in density occurs.

In a system of constant space density, with the luminosity function everywhere the same and no loss of light because of absorption, the ratio of the numbers of stars in successive intervals of magnitude is 3.98. If the star ratio is less than this limit, the average density decreases. Counts from the *Draper Catalogue* of stars brighter than $m=8.0$ give star ratios which show surprisingly little dependence on galactic longitude. Stars in the Carina-Sagittarius region are more numerous than those in the opposite part of the sky, but the maximum ratio, even in the direction of the center of the stellar system, scarcely exceeds 3.0. This result agrees well with an extrapolation of the data used in this discussion. Thus the values of log Q_ξ, combined with the mean log N_m for all longitudes ($\beta=0°$), lead to star ratios for the first 5 or 6 mag., which, in the directions $\lambda=L$ and $L+180°$, average 3.0 and 2.8, respectively.

Moreover, the decreasing density which is thus clearly indicated sets in, in all directions, at no very great distance from the sun. Let R be the star ratio for magnitudes $m+1$ and m, and ρ the geometrical mean distance of all the stars of these magnitudes; further, let δ represent the relative density in two concentric shells having radii defined by log $\rho-0.2$, log ρ, and log $\rho+0.2$. It is then not difficult to show that δ is approximately equal to $R/3.98$. For stars of the fourth and fifth magnitudes the geometrical mean distance is roughly a hundred parsecs. The density at 125 parsecs (log $\rho=2.1$) should therefore be approximately $3.0/3.98=0.8$ that at 80 parsecs (log $\rho=0.9$). In the same way, the star ratio for the third and fourth magnitudes indicates a similar decrease in density between 50 and 80 parsecs. In other words, we have a notable drop in density beginning at a point close to the sun.

It is doubtful, however, if the density near the sun really decreases as rapidly as the argument based on star ratios would indicate—indeed, excellent evidence indicates that it does not. There is little to fear from the assumption concerning absorption losses; but the other supposition, namely, a constant luminosity function, may not be justified. In fact, it is likely that the neighboring local cluster of B stars appreciably alters the frequency distribution of luminosity near the sun. The occurrence of an exceptional number of very luminous stars in this region would swell the numbers among

the brighter apparent magnitudes, and hence give a star ratio smaller than that to be expected on the basis of the actual density distribution. But, even so, the evidence that the solar system is at or near a point of maximum density is conclusive.

Some modification of the original hypothesis of a simple spheroidal system is therefore demanded; and that required is obviously the introduction of the local cluster. The modification, moreover, must be so radical that the space density, instead of increasing toward the center of the larger system as indicated in Figure 6, will begin to decrease almost at once, in all directions, as we proceed from the solar system outward into space. Through an independent line of argument, we thus reach again the conclusion that the local cluster is the chief factor in determining the space density in the neighborhood of the sun. At the same time, the quantity ξ loses its meaning as the co-ordinate of a central point of maximum space density in the galactic system as a whole. The center itself disappears in the perspective of the blended systems and must be discovered by other means.

The preceding discussion, however, is not altogether barren of numerical results. When only a single limiting magnitude is considered, the surface distribution of the stars behaves as though the original hypothesis were valid, and this hypothesis may be used, as it has been used, to obtain interpolation formulae for the derivation of good average values of the ratios of surface density. These, in turn, have been combined by a method which is independent of the hypothesis to give the ratio of total light, which by (59) is directly the ratio of the integrated densities, and, in consequence, an important result. Although the various ratios for the directions of the center and the anti-center cannot be used in the manner originally contemplated, they provide means for a tentative determination of the space density in these directions and a valuable control on the results.

11. SPACE DENSITY IN THE DIRECTIONS OF CENTER AND ANTI-CENTER

Notwithstanding uncertainties mentioned in the introduction, the only hope of disentangling the local cluster from the larger system seems to lie in the direct investigation of space density. The

simplest method of discussing the present data is that originally proposed by Schwarzschild.[1]

The most striking differences in space density should appear in the directions of the center and anti-center. Good values of the surface density for these directions may be found by combining log Q_ξ with the mean distribution, in accordance with equations (55). The results lead to the following quadratic exponentials for the derivatives of N_m:

$$\left.\begin{aligned} \log\frac{dN'_m}{dm} &= -5.89+0.748\,m-0.0288\,m^2 \quad (\lambda=L) \\ \log\frac{dN''}{dm} &= -5.36+0.632\,m-0.0230\,m^2 \quad (\lambda=L+180°) \end{aligned}\right\} \qquad (61)$$

To obtain the densities, these expressions must be combined with the luminosity function for photographic magnitudes.

This function has been determined by van Rhijn[2], who finds, in agreement with *Contribution* No. 273,[3] that the distribution of stellar luminosity cannot be completely represented by the gaussian curve which Schwarzschild's theory presupposes. The failure begins at absolute magnitude $+7$ or $+8$ and is in the direction of frequencies for stars of low luminosity which are much too small. To meet the requirements of the theory, this defect must be disregarded. Space densities thus calculated will decrease too slowly with increasing distance, both toward the center and the anti-center, but the relative values of the density in the two directions should be little affected.

A rough calculation based on van Rhijn's data gives the gaussian function for luminosity:

$$\log \phi(M) = \text{const.}+0.18\,M-0.028\,M^2\,. \qquad (62)$$
$$(M=m-5\log\rho)$$

Equations (9), *Contribution* No. 229,[4] then give for the space density

$$\left.\begin{aligned} \log\Delta(\rho) &= -4.06+3.03\,(\log\rho)-0.565\,(\log\rho)^2 \quad (\lambda=L) \\ \log\Delta(\rho) &= -1.21+1.42\,(\log\rho)-0.389\,(\log\rho)^2 \quad (\lambda=L+180°) \end{aligned}\right\} \qquad (63)$$

[1] *Astronomische Nachrichten*, **190**, 361, 1912.

[2] *Groningen Publication*, No. 38, 1925. The values used are from the last column of Table 71.

[3] *Astrophysical Journal*, **59**, 311, 1924. [4] *Ibid.*, **55**, 242, 1922.

These expressions have maxima for log $\rho = 2.70$ and 1.86, respectively, the corresponding distances being 500 and 70 parsecs. The constants are such that the density is unity at 500 parsecs in the direction of the center. For distances less than those corresponding to maximum density the formulae are not applicable, but the probable form of the curve near the sun may be found with the aid of

TABLE VII*

SPACE DENSITY IN GALACTIC PLANE

Log ρ	ρ	$\Delta(\rho)$	
		$\lambda = L$	$\lambda = L + 180°$
	Parsecs		
0.0	1	1.00	1.00
2.0	100	0.98	0.93
2.3	200	.95	.79
2.5	316	.90	.63
2.7	500	.84	.47
3.0	1000	.70	.28
3.3	2000	.49	.17
3.5	3162	.33	.08
3.6	3981	.27	.06
3.7	5012	.20	.04
3.8	6310	.15	.03
3.9	7943	.11	.02
4.0	10,000	.08	.014
4.1	12,590	.06	0.01
4.2	15,850	.04	....
4.3	19,950	.03	....
4.4	25,120	0.02	....

* The densities in this and the following table probably decrease too slowly. They have been computed with a gaussian distribution of absolute magnitudes, which assigns too small a frequency to stars of low luminosity. Relative values in different directions should be little affected.

data obtained incidentally in calculating the luminosity function. Van Rhijn's results[1] for latitudes 0°–20°, which are averaged values for all longitudes, give the mean density at 500 parsecs as 0.55 of the density near the sun. Consideration of similar results for other latitudes leads to the adoption of 0.63 as the value for the galactic plane itself. At a distance of 500 parsecs in the direction of the center and the anti-center the foregoing formulae give 1.00 and 0.60, respectively. Their mean is 0.80; hence the density at the sun, in the same unit, is $0.80/0.63 = 1.27$.

[1] *Groningen Publication*, No. 38, p. 15.

These results are listed in Table VII and shown graphically in Figure 8; the unit, however, has now been changed so that the density at the sun is equal to 1.00. In spite of the inadequacy of the theoretical relations used in deriving these values of the density, there can be no doubt as to the general features of the distribution. The striking asymmetry and the sharp maximum near the sun are in full agreement with the results already found by other means.

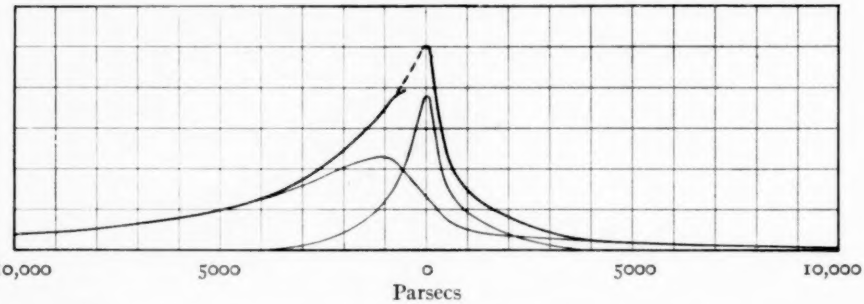

FIG. 8.—Smoothed space density (ordinates) calculated from surface densities with the aid of a gaussian luminosity function. The heavy curve, with a maximum very near the sun, indicates the distribution in the directions of center (left) and anti-center (right). The component curves represent the contributions of the local cluster, supposed to be concentric with the sun and nearly symmetrical, and of the larger system. The adopted resolution gives regularity to the larger system, and, to the local cluster, properties confirmed by other evidence.

The asymmetry of the curve for the larger system is attributed to the circumstance that the curve really corresponds to the density distribution in the direction, not of the center itself, which is hidden by obscuring clouds, but of the branches of the Milky Way, inclined some 6° or 8° to the galactic plane. In a symmetrical system resembling a highly-resolved spiral nebula, the density distribution along a line thus inclined would be asymmetrical about a maximum less distant than the geometrical center, and hence similar to that found for the larger system. Any selective scattering would increase the similarity by shifting the apparent maximum of density toward the sun.

Moreover, the degree of asymmetry is in harmony with the adopted ratio of total light, $B_\xi = 3.04$. A graphical integration of the densities in the direction of center and anti-center gives a density ratio of 3.0, which shows that the use of a gaussian curve for the luminosity function has produced no relative distortion of the two branches of the curve. It should be emphasized, however, that, because of this procedure, both branches probably indicate too slow a decrease in density with increasing distance; and, further, in accordance with

the introduction, that the regularity of the curve is no criterion of the absence of large fluctuations in space density, especially at great distances.

12. MEAN DENSITY PERPENDICULAR TO THE GALACTIC PLANE

To indicate the behavior of space density perpendicular to the galactic plane, mean values for $\beta = +90^\circ$ and -90° have been calculated by the method described above. The resulting formula is

$$\log \Delta(\rho) = -0.48 + 1.160 \, (\log \rho) - 0.452 \, (\log \rho)^2 \, , \tag{64}$$

in which the unit is the same as in (63). The numerical results, reduced to unit density at the sun with the aid of the factor used for (63), are in the third column of Table VIII. The last column of this

TABLE VIII

MEAN SPACE DENSITY PERPENDICULAR TO GALACTIC PLANE

Log ρ	ρ	$\Delta(\rho)$ Eq. (64)	$\Delta(\rho)$ van Rhijn*
	Parsecs		
0.0	1	(1.00)	1.00
1.9	79	0.90	0.91
2.0	100	.84	....
2.1	126	.73	.79
2.3	200	.50	.55
2.5	316	.31	.34
2.6	398	.24	....
2.7	500	.17	.16
2.8	631	.13	....
2.9	794	.09	0.04
3.0	1000	.07	....
3.1	1259	.05	....
3.2	1585	.03	....
3.3	1995	.02	....
3.4	2512	0.01	....

* Extrapolations to 90° of values given by van Rhijn for 0°–20°, 20°–40°, and 40°–90°.

table gives values extrapolated from van Rhijn's results.[1] The general average agreement of these two columns checks the zero-point reduction used for (63). Equation (64) is subject to the same limitations as (63); but, in conjunction, these formulae give a general indication of the relative decline in space density in and perpendicu-

[1] *Loc. cit.*

lar to the galactic plane. For these directions the diameters of the surfaces of constant density have a ratio which for densities of 0.02–0.2 seems to be nearly constant and equal to 7.5. With increasing density the ratio decreases to about 2.8 for a density of 0.9.

13. SOME CONCLUSIONS

The smoothed distribution of density illustrated in Figure 8 reveals more clearly the difficulty involved in locating the center of the system. Any maximum of space density at the center, if relatively near, is masked by the local cluster, or, if distant, may fail to appear for any one of a variety of reasons, among them the general smoothing-out of density inherent in the method of calculation. The contours of equal density give little information as to the center, because those corresponding to large or moderate densities are influenced by the local cluster, while those for small densities are too unreliable to be useful.

The approximate symmetry of the nucleus of bright helium stars, clustered about a point less than a hundred parsecs distant,[1] suggests for the local system as a whole a spheroidal form, nearly concentric with the sun. An attempt to resolve the density distribution in the directions of center and anti-center into two symmetrical curves, one representing the local cluster, the other the larger system, shows that no such resolution is possible. Nor is it possible from the present data to find for the larger system any symmetrical curve which does not require a very improbable distribution of density for the local cluster. The adoption of a symmetrical, or nearly symmetrical, curve for the local cluster gives, however, an asymmetrical but not impossible succession of densities for the larger system. The results in this case are not unique, but it is more than significant that a resolution which leads to much regularity in the larger system is one which assigns to the local cluster characteristics already recognized from other data—a 3:1 ratio of cluster and non-cluster stars near the sun, an equal mixture, on the average, at distances of 700 parsecs, and a radius for the cluster of some 3000 or 4000 parsecs.

Any attempt to assign much smaller dimensions to the local cluster leads to serious difficulty with the larger system. If we sup-

[1] Charlier, *op. cit.*, No. 14; Gerasimovič, *Vierteljahrsschrift*, **61**, 219, 1926.

pose the diameter of the local system to be small, but retain the large space density indicated by the data, the curve for the larger system will show maxima at distances of a few hundred parsecs in the directions of both center and anti-center, and, in fact, in all other longitudes as well. The density surface of the larger system would thus show a ring of maximum density surrounding the sun. If both diameter and content of the cluster are supposed to be small, the resulting curve for the larger system will have a single maximum near the sun similar to the curve of total density shown in Figure 8. The adopted dimensions, although only a rough estimate, are such as to avoid these difficulties.

The curves corresponding to the provisionally accepted resolution are shown in Figure 8. An increase in the slight asymmetry adopted for the local cluster would displace the maximum for the larger system from its position at 1000 parsecs, to a distance of 2000 parsecs or more; but whatever the exact location of this maximum, the space density of the larger system apparently increases for some distance in the direction of the center to a point of high density near that previously identified with the center itself. Now, however, it seems improbable that this maximum should coincide with the geometrical center. The form of the density curve for the larger system, uncertain as it is, is against such a coincidence. Although the contours of equal density are not even approximately concentric, those for small densities agree in indicating that the geometrical center is at a much greater distance than the neighboring maximum.

Aside from the local cluster, therefore, the data seem to indicate a system of great size, with its geometrical center at a distance of several thousand parsecs and notable in that the center appears to be undistinguished by any peculiarity of space density. This, however, is opposed to the whole trend of modern cosmological thought, which is toward parallelism of structure in the stellar system and in spiral nebulae, and thus seems to require for the stellar system both symmetry and a central condensation of relatively high density.

Circumstances connected with the density problem show that other and even more important maxima than that found may exist at great distances from the sun. At the same time, however, the component curve tentatively ascribed to the larger system indicates

that beyond one or two thousand parsecs the smoothed density decreases continuously with increasing distance. It is difficult to reconcile this result with the existence of a more distant maximum of such importance that it could be regarded as the central condensation of the system, at least not without specification of unusual conditions. Such a condensation, together with scattered stars and star clouds distributed over the galactic plane, might indeed give a smoothed density function similar to that found, but only in case the spacing of the various aggregations over the galactic plane were very peculiar and highly improbable.

Nor does it help to suppose that the central condensation is so distant that the stars composing it are too faint to be seen; the anomalous asymmetry and the decrease in the smoothed density of the stars actually observed would still remain. Closely allied is the attempt to find an explanation in changes in the luminosity function with increasing distance from the center. Certain spirals suggest such a possibility; but the evidence, such as it is, reveals no change outside the central condensation, and, as before, leaves unexplained the decrease in space density among the outlying stars.

These rather speculative considerations are mentioned chiefly as details which may contribute to, or modify to some extent, what seems probably to be the most important factor involved, namely, the influence of obscuring material. Thus far the effects of obscuration and absorption have been completely ignored, because no data are available for treating them quantitatively. But obscuring material is certainly present, both in spirals and in our own system; and in many spirals, at least, it must hide the central condensation, in part if not wholly, from outlying points in the plane of the nebula. It has more than once been suggested that the two branches of the Milky Way, separated by obscuring clouds, as apparently they are, present a phenomenon analogous to what is observed in almost every edge-on spiral that we know; and it seems probable that close analogy really exists.

With the acceptance of this suggestion, the peculiar position of the maximum of space density finds a simple explanation in harmony with the general trend of cosmological evidence: As in the spirals, the central condensation of the stellar system is hidden behind

obscuring material, concentrated close to the galactic plane. Scattered stars and aggregations of stars seen above and below the obscuring clouds form the two branches of the Milky Way. The surface densities adopted for 0° latitude in the general direction of the center are extrapolations downward to the galactic plane of densities observed in the branches. These extrapolations carry with them the characteristics of space density in the directions of the branches, which are inclined some 6° or 8° to the fundamental plane. In analogy with spirals, the surfaces of equal space density in the larger system are very much flattened; the calculated distribution of density therefore differs greatly from the actual distribution in the direction of the center. Instead of being symmetrical, it rises rapidly to a moderate maximum, much less distant than the center, beyond which it declines slowly. Any absorption or scattering, which may well occur in the vicinity of obscuring clouds, must accentuate the asymmetry and shift the maximum to a point still nearer the sun.

This explanation accounts well for the peculiarities of the calculated distribution, but leaves the distance of the center of the system wholly undetermined. At the same time, it removes the conflict between certain observational results and the center found by Shapley from the globular clusters, which he placed at a distance of about 20,000 parsecs in longitude 325°.[1] The agreement of the longitude with that derived from the star counts is close, and any further hesitation in accepting Shapley's value of the distance arises chiefly from a feeling that the ill-defined and widely scattered collection of a hundred or so clusters may not be closely concentric with the system of the stars.

There remains still a question as to the part played by the local cluster, which seems to have an importance not hitherto suspected. Its central density is not only of dominating influence near the sun, but is nearly twice that of the neighboring maximum of the larger system. Further, its total density in the direction of center and anti-center is about one-third the similar total for the larger system. This last figure, however, may give an exaggerated notion of the probable number of stars involved in the cluster unless the volume

[1] *Mt. Wilson Contr.*, No. 157; *Astrophysical Journal*, **49**, 311, 1919.

factors corresponding to the different densities are taken into account. Since the distribution of space density for the cluster in directions perpendicular to the galactic plane is unknown, the simplest method of trying to estimate the content of the cluster is to assume that for any magnitude the number of cluster and non-cluster stars is such that the weighted mean values of the parameters L, p, and L_0 are those shown in Figure 5. The totals thus found are 60,000,000, 100,000,000, and 20,000,000, respectively. The method, admittedly, is of the roughest kind, but sufficient to indicate that the members of the cluster are to be numbered in many millions.

In seeking for analogies that may illustrate the connection of the local cluster with the primary system, the larger aggregations of stars distributed along the arms of the highly resolved spiral nebulae immediately suggest themselves. The dimensions of these groups relative to the systems in which they occur seem, however, to be rather small to make the analogy close. Both the high density and the large diameter of the cluster suggest something rather exceptional. In size, perhaps the secondary condensation in Messier 51 is a closer parallel, although in other respects, important differences are obvious. Thus the condensation is at the end of one of the arms and on the very edge of the system; and its reddish color[1] suggests physical relationship with the central region of the nebula, rather than with the smaller aggregations scattered along the arms, in which the stars of high temperature and great luminosity seem to congregate. In neither of these particulars is there good agreement with the properties of the cluster. Finally, the numerous instances of closely associated nebulae perhaps should not be overlooked—instances far too numerous to be accounted for wholly by chance coincidences in direction. Those pertinent for close comparison must show a small object closely involved in one very much larger.

It would be saying too much to insist that the local cluster in its relation to the larger system finds a close analogue in any known object; but in tracing parallels it is at least to be noted that the existence of an important subsystem, having its own characteristic properties, is not an isolated phenomenon.

[1] *Mt. Wilson Communication*, No. 36; *Proceedings of the National Academy of Sciences*, 2, 553, 1916.

The prima facie evidence of star counts suggests a simple hypothesis—an eccentric location of the sun within a stellar system having spheroidal symmetry. The main features of the distribution of surface density for any limiting magnitude may thus be accounted for. The orientation of the spheroid and the direction of its center vary, however, with the magnitude. This points to the existence of a large local system of high density, concentric with the well-known cluster of helium stars. Attempts to ignore the local system only bring it more clearly into view and confirm the conclusion that it contributes a large fraction of the space density near the sun. At the same time, it appears that the total space density must decrease, on the average, in all directions with increasing distance from the sun. Calculation proves that the maximum is at or very near the sun, and shows that the apparent distribution of total density in the directions of center and anti-center is very asymmetrical and cannot be the resultant of two symmetrical systems. A nearly symmetrical local system, having properties agreeing with those indicated by independent evidence, leaves for the larger system an asymmetrical distribution of space density, with its geometrical center at a much greater distance than the only maximum visible in the curve of smoothed density. This result can only be regarded as anomalous and is attributed to the influence of obscuring material, which is known to be present. Allowance for the effect of diffuse nebulosity on surface and space densities avoids the anomaly and affords a means of reconciling, qualitatively at least, the data on stellar distribution with the analogies traced in recent years between the galactic system and spiral nebulae. Except for the presence of the local cluster, the analysis therefore leaves the sun eccentrically situated within a symmetrical system—the larger system; but the symmetry is now that of a spiral nebula, which is quite different from that superficially indicated by the star counts.

As a directive aid to further research, the preceding results, combined with others from various sources, lead to the following picture of the probable structural relations in the stellar system: The galactic system is a vast organization resembling Messier 33, although probably larger and perhaps even more completely resolved

into stars. It includes a central condensation, scattered stars, and aggregations of stars distributed over the galactic plane, the large groups of the Milky Way corresponding to the knots and condensations of the spiral arms of the nebula. An outlying aggregation of exceptional size and density, but having the physical characteristics of condensations in the nebular arms in that it includes giant stars of high temperature, accounts for the local cluster. Like the spirals, the system also includes diffuse nebulous material concentrated near the galactic plane, dark and obscuring, or luminous if stimulated to shine by the radiation of neighboring stars of high temperature.

The central condensation, in longitude 325°, is hidden behind the dark nebulosity which marks the great rift in the galaxy. Stellar aggregations, seen above and below the obscuring clouds in the manner familiar in edge-on spirals, form the two branches of the Milky Way, extending from Cygnus to Circinus.

The diameter of the system is large—80,000 to 90,000 parsecs, if it may be regarded as coextensive with the system of globular clusters; at least 80,000 parsecs, if the absolute magnitudes of normal B0–B2 stars may be assigned to the faint blue stars of the Milky Way; and at least 60,000 parsecs, if, as seems probable, stars in the direction of the anti-center occur at distances as great as 10,000 parsecs.

The sun is situated almost exactly in the galactic plane defined by faint stars, but a little to the north of that determined by Cepheids, faint B stars, and some other special classes of objects. Its distance from the central condensation is perhaps 20,000 parsecs, but less than 100 parsecs from the center of the large aggregation which constitutes the local system.

The local system, assumed to be spheroidal, is centered in longitude 230°± and is 40–50 parsecs south of the galactic plane. Its diameter is 6000 or 8000 parsecs, and it includes many millions of stars. Its principal plane, which is well defined by the much-flattened nucleus of bright helium stars (Gould's Belt), and to a lesser extent by diffuse nebulosity, is inclined 12° to the plane of the Milky Way clouds.

The space density of the larger system presents many irregularities, but, on the average, increases toward the central condensation. The local system is more homogeneous, and notable in that its

contribution to the space density near the sun is three times that of the larger system. At points in the galactic plane distant about 700 parsecs the average contributions of the two systems are equal.

The implications of this amplified hypothesis agree well, in a qualitative way at least, with the observed facts of stellar distribution. By way of clarification, the correspondences may be briefly summarized: Viewed from within, the stars of both systems congregate toward their respective fundamental planes. To any given limiting magnitude the observed surface distribution is a weighted mean which depends on the relative numbers of stars contributed by each system. Since a large majority of stars brighter than the sixth magnitude belong to the local system, the direction of maximum surface density and the position of the galactic pole for these stars are very nearly those of the nucleus of bright helium stars which defines the plane of the local cluster. On the other hand, relatively few stars fainter than the fifteenth magnitude belong to the cluster. The orientation of the pole and the direction of the center for faint limits are therefore determined by the characteristics of the larger system. Between these limits the parameters L, p, and L_0 vary progressively with increasing limiting magnitude.

Obscuring clouds conceal the great surface density which otherwise might be expected in the direction of the center of the larger system. Except for local fluctuations, the maximum surface density is that observed in the branches of the Milky Way in the longitude of the center. The variation of surface density with longitude depends primarily on the very eccentric position of the sun, and, although more complicated than the simpler case discussed above, still has as its principal term a first harmonic. Since the local system is nearly concentric with the sun, its contribution to the total number of stars per square degree in a given latitude is practically the same in all longitudes. The amplitude of the variation in $\log N_m$ is therefore affected by the presence of the local cluster, but its general character for the combined systems is essentially the same as for the larger system alone. Approached from the observational standpoint, therefore, the distribution of surface density in the system proposed, to any limiting magnitude, has the characteristics of a

simple spheroidal system; but owing to the varying percentage of stars contributed by the two systems, the characteristics of the spheroid vary with the magnitude.

The great dispersion in absolute magnitude distributes fluctuations in space density over a wide range in apparent magnitude. The surface density therefore shows the continuous increase with increase in the limiting magnitude generally observed; and the difficulty of interpreting this result in terms of fluctuating space density leads to the acceptance of the usual smoothed curve of continuously decreasing space density. The distribution usually accepted for the direction of the center of the larger system is really the quite-different distribution corresponding to a direction inclined a few degrees to the galactic plane. As already explained, this must be asymmetrical, with a nearby maximum whose distance is perhaps further diminished by the influence of scattering. Finally, the presence of the local cluster with its relatively high space density brings the maximum into the immediate vicinity of the sun, so that the calculated change in space density is a decrease in all directions—slowest in the direction of the center, more rapid toward the anti-center, but most rapid of all, because of the greatly flattened form of the system, in directions perpendicular to the galactic plane.

Carnegie Institution of Washington
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THE PECULIAR BRIGHT-LINE SPECTRUM OF RY SCUTI[1]

By PAUL W. MERRILL

ABSTRACT

Bright hydrogen lines in RY Scuti were discovered at Mount Wilson in 1921. Spectrograms taken from 1921 to 1927 show a well-marked bright line in the position of the *nebular* line λ 4658. Several fainter lines are apparently associated with it. Their wave-lengths and intensities are 4658.08 (4), 4701.5 (2), 4733.6 (1+), 4755.0 (1), 4770 (1).

A number of bright-line stars observed at Mount Wilson[2] have been selected for special spectroscopic study. These include (*a*) spectra having unusual bright lines, (*b*) spectra in which the bright lines have some peculiarity of structure, (*c*) those in which changes are occurring, (*d*) spectroscopic binaries.

One of the most interesting spectra exhibiting unusual bright lines is that of RY Scuti.[3] This spectrum is the only one of its kind known to the writer. Its most remarkable feature is the presence of the nebular line λ 4658. Bright hydrogen lines were discovered in this spectrum in 1921, in the Mount Wilson search for Be stars.[4] Objective-prism photographs by Mr. Humason and the writer show a bright $H\alpha$ line on the following dates: 1921 August 1; 1925 May 17 and June 17; 1926 May 17; 1927 July 4. The line is very strong compared to the neighboring continuous spectrum.

Slit spectrograms taken with a one-prism spectrograph attached to the 100-inch telescope are listed in Table I.

All plates except the third were taken with the 18-inch camera. The visual magnitude was about 9.5 on all the dates. The color appears yellower than that of a typical B-type star. The usual exposure of about four hours yielded plates moderately well exposed

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 349.

[2] The program includes Be stars discovered here, and a few, previously known, for which detailed descriptions have not been published.

[3] RY Scuti, H.D. 169515, B.D. $-12^{\circ}5045$; R.A. 1900 $18^{h}\ 19^{m}9$; Dec. 1900 $-12^{\circ}\ 45'$; mag. var. Remark in H.D.: "Max. 8.3; min. 9.2. Class and period, unknown. The spectrum is nearly continuous. Very faint, dark hydrogen lines are seen, and bright lines are suspected to be present."

[4] *Publications of the Astronomical Society of the Pacific*, **34**, 351, 1922; *Mt. Wilson Contr.*, No. 294; *Astrophysical Journal*, **61**, 389, 1925.

from $H\beta$ to $H\gamma$, but the spectrum to the violet of $H\gamma$ is weak on nearly all the photographs. Considerably longer exposures would be required to give proper density to the continuous spectrum from $H\gamma$ to $H\delta$.

Bright $H\beta$ is an outstanding feature on all the Mount Wilson slit spectrograms. When the exposure was sufficient, $H\gamma$ appears as a well-marked bright line with a faint dark border on the violet side. Inasmuch as it is stated in the *Henry Draper Catalogue* that "very faint dark hydrogen lines are seen," it seems clear that the

TABLE I

OBSERVATIONS OF RY SCUTI

Plate	Date	Velocity	Weight	Remarks
		km/sec.		
C 1255	1921 Aug. 24	+22	1.5	Under-exposed
1260	Aug. 25	26	3	
1272	Sept. 8	....	....	7-inch camera; strong exposure
1710	1922 May 17	34	2.5	
2364	1923 July 28	28	2	
2973	1924 Aug. 21	31	1	Under-exposed
3382	1925 July 6	28	3	
3807	1926 May 29	46	2.5	
4275	1927 May 15	+20	1.5	
Mean......		+30.0	17.0	

character of the hydrogen lines must have changed since the Harvard observations were made. No appreciable changes have occurred during the Mount Wilson series from 1921 to 1927.

The lines available for radial-velocity determinations are $H\beta$, $H\gamma$, and λ 4471, *He*. These are narrow bright lines, but on several plates the photographic density at $H\gamma$ and λ 4471 is insufficient for the most accurate measurement. These two lines are accompanied by weak absorption on their violet sides.

The third and fourth columns of Table I give the measured velocities and their weights. Unit weight is accorded each good line.

If all plates be given the same weight, the mean velocity becomes +29.3 km/sec.; if each line be given the same weight, the result is +30.2 km/sec. It is thus seen that the final velocity is nearly independent of the system of weighting. The velocities from the various lines are as follows:

	No. Plates	Weight	Mean Velocity
$H\gamma$	6	4.0	+33.2
λ 4471	7	5.5	26.4
$H\beta$	8	7.5	+31.0
Unweighted mean			+30.2

Besides the lines of hydrogen and helium, the spectrum shows a group of lines in the neighborhood of λ 4700 whose chemical origin has not been identified. Their wave-lengths computed with the aid of the radial velocity derived from $H\beta$, $H\gamma$, and λ 4471 are given in Table II. For this calculation individual velocities for each plate, as given in Table I, may be used, or the mean value, +30.0 km/sec.,

TABLE II

BRIGHT LINES

I.A.	P.E.	Int.	No. Plates	Wave-Number	Differences
4658.08	±0.03	4	8	21462.1	
					198.2
4701.5	.08	2	8	21263.9	
					144.2
4733.6	±0.1	1+	6	21119.7	
					95.1
4755.0	....	1-	3	21024.6	
					66
4770	....	1	3	20958.5	

for all plates. The first method tends to reduce the effect upon the unknown lines of such systematic plate errors as enter into the relationship between the stellar spectrum and the comparison; the second method reduces the effect of accidental errors of measurement. As a test, I have computed the wave-length of λ 4658, the most accurately measurable of the unknown lines, by both methods. The first gives 4658.09±0.030; the second, 4658.08±0.028. The close agreement of the probable errors shows that the two sources of error enter about equally. Probably both are reasonably small compared with the accidental errors of measurement of the line itself. For the other bright lines, the mean velocity +30.0 km/sec. has been used.

These lines are organized in spacing and intensity somewhat like a line series, but converge toward longer instead of shorter wave-lengths. The frequency-differences are roughly proportional to the numbers 4, 3, 2, 1. The first two lines are well-marked bright lines.

The others are weaker and stand out less clearly from the background formed by the continuous spectrum.

In addition to the lines in Table II, numerous weak maxima, at the limit of visibility, occur in the continuous spectrum. Under the microscope these are so uncertain as to defy measurement. There may be a very faint maximum near the position of the chief nebular line λ 5007.

In view of all the circumstances, it seems highly probable that the line λ 4658.06 is identical with the line near this wave-length observed by Wright in nebulae,[1] and by the writer in R Aquarii.[2] The arguments leading to this conclusion are as follows: (1) The wave-lengths agree within errors of measurement. (2) The line is associated with bright hydrogen and helium lines in RY Scuti, as in nebulae. (3) No emission line has been observed at this wave-length in any celestial object except nebulae and their nuclei. (4) At several minima the spectrum of R Aquarii has been essentially that of a planetary nebula, and has shown the bright line λ 4658. On certain occasions when this line was relatively strong, a weaker line at λ 4701 was present, as in RY Scuti. (5) The high residual radial velocity of RY Scuti, +45 km/sec., is appropriate for a planetary nebula.

Thus, aside from the somewhat doubtful presence of N_1, we find strong reasons for believing RY Scuti to be related to planetary nebulae through the line λ 4658. It therefore seems justifiable to add the last four lines in Table II to the list of nebular lines. λ 4701.5 is probably associated in origin with λ 4658.08, and this may well be true of the remaining three lines.

RY Scuti appears to belong to that small group of objects in which nebular lines ordinarily of minor importance surpass in intensity the green lines N_1 and N_2. These unusual objects offer special opportunities for classifying nebular lines according to their chemical origin or to the excitation required for their production.

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[1] *Publications of the Lick Observatory*, **13**, 193, 1918.

[2] *Publications of the Astronomical Society of the Pacific*, **34**, 134, 1922.

MINOR CONTRIBUTIONS AND NOTES

THE EFFECTIVE AMOUNT OF WATER-VAPOR IN THE ATMOSPHERE AT THE ECLIPSE OF JANUARY 24, 1925

ABSTRACT

The authors' estimate of 1 cm of precipitable water in the atmosphere, at the time of the eclipse, is found to be about twice that calculated from hygrometric readings at Hartford. The conclusions of the authors would, therefore, not be appreciably affected.

Stetson, Coblentz, Arnold, and Spurr[1] have computed the depth of the equivalent horizontal layer of precipitable water in the atmosphere from Hann's formula, $d=2.3e$, where d equals the depth in millimeters of a horizontal layer of water equivalent to all the water-vapor overhead and e is the vapor pressure in millimeters. When applied to the absorption of radiation from celestial bodies, the amount of water-vapor in the optical path must be used. This is given by

$$(d)=2.3e \sec z ,$$

where z is the zenith distance of the observed body. At Middletown the value of sec z for the sun was 3.29. For the value $e=0.58$, found by them from meteorological observations made at Hartford, Connecticut, the thickness of precipitable water affecting the observations was 4.4 mm instead of 1.33 mm. Our estimate[2] of 1 cm for Middletown was, therefore, about twice the amount calculated from atmospheric conditions at Hartford, instead of eight times that amount, as stated in the paper referred to above. This discrepancy of about 50 per cent, as mentioned in our discussion, would scarcely affect our results.

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[1] *Astrophysical Journal*, **66**, 84, 1927 (September).

[2] *Ibid.*, **64**, 140, 1926.

NOTE ON WAVE-LENGTHS IN THE NITROGEN PEROXIDE ABSORPTION SPECTRUM

A search of *Science Abstracts* and of *Chemical Abstracts* not showing any record of measurements of wave-lengths in the absorption spectrum of nitrogen peroxide, it was thought worth while to make some measurements by photographic methods, since the best earlier measurements, according to Kayser's *Handbuch der Spectroscopie*, were those made by Hasselberg in 1878 by visual methods, with prisms.

The present photographs were made with a 21-foot Rowland concave grating. The continuous spectrum was furnished by the right-angled carbon arc or by the Tungsarc, and the comparison spectrum by the standard (Pfund) iron arc.

Nitrogen peroxide was made by heating Baker's analyzed lead nitrate in a specially constructed all-glass generator, of which the cell formed a part which was sealed off after the

$$NO_2 \rightleftarrows N_2O_4$$

had been collected in it. Some cells contained both vapor and liquid; others, vapor only, sealed off at a known temperature. Precautions were taken to get rid of water-vapor, dust, air, and oxygen. The length of path of the light in the gas varied from 0.4 to 8.0 cm.

One to four measurements were made on each of about sixty-seven hundred lines on photographic plates in the first order (one plate in the second order). Tables give the number of plates on which a line was measured, its estimated intensity, its wave-length, and the average deviation from the mean. This average deviation is 0.03 A; hence six figures are given as significant. Wave-lengths are given from 3978.67 to 6323.22 A.

On account of expense of publication, the tables have been published privately. A copy will be gladly furnished gratis to anyone interested. Address: Rouss Physical Laboratory, University, Virginia.

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UNIVERSITY OF VIRGINIA
November 3, 1927